UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics M255: Linear Programming and Optimization

COURSE CODE	:	**MATHM255**
UNIT VALUE	:	**0.50**
DATE	:	**13-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0972-3-170

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. The following linear program P is in standard form.

$$\begin{array}{lrcl} \text{Maximise} & x_1 + 3x_2 & & \\ \text{subject to} & 2x_1 + x_2 - x_3 & \leqslant & 6 \\ & x_1 + 2x_2 + 2x_3 & \leqslant & 5 \\ & -x_1 + x_2 + x_3 & \leqslant & 1 \\ & x_1, x_2, x_3 & \geqslant & 0 \end{array}$$

   Bring it to equality form and solve it by the simplex algorithm. Identify the basic feasible solution after each pivot step.

   Assume the last inequality is changed to $-x_1 + x_2 + x_3 \leqslant 1 + t$ where t is a small number. What is the new optimal value of P?

2. Let x, c be vectors in $\mathbb{R}^n$, A an $m \times n$ matrix and b a vector in $\mathbb{R}^m$. Let P be the linear program:

$$\begin{array}{rcl} \text{Maximise } c^T x & & \\ \text{subject to } Ax & \leqslant & b \\ x & \geqslant & 0 \end{array}$$

   Show that the feasible set of P is convex.

   Write down dual of P. What is Complementary Slackness?

   State and prove the Weak Duality Theorem for P and its dual.

   State the Strong Duality Theorem for P and its dual.

3. Explain what is meant by a two-person zero-sum game with payoff matrix A.

   Give the definition of *pure strategy*, *randomised strategy*, *equilibrium*, and *saddle point*.

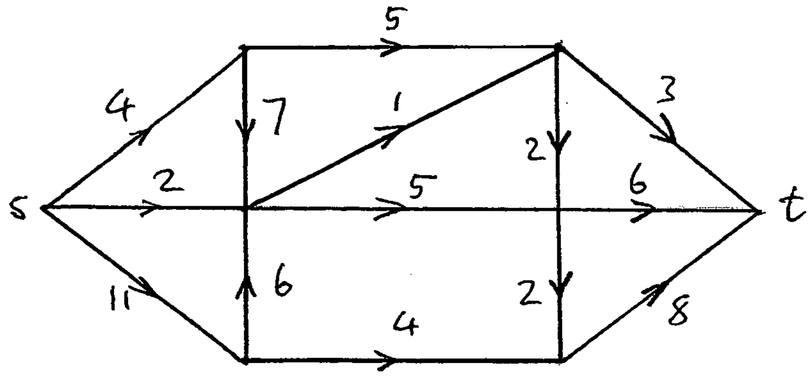
   Show that if a game has a saddle point then there is an equilibrium consisting of pure strategies.

   Find the equilibrium for the game with payoff matrix

$$\begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

4. Prove that in a network with capacities, there is a cut whose value is equal to that of the maximum flow.

Find a maximum flow in the following network.



Explain why the Ford-Fulkerson algorithm always terminates in finitely many steps in a network with integer capacities.

5. Formulate the transportation problem.

Write down the dual of the transportation problem.

Explain how to check whether a particular transportation arrangement is optimal.

Find the optimal transportation for the array

	D_1	D_2	D_3	
S_1	3	2	6	7
S_2	5	4	4	8
S_3	2	5	7	9
	10	11	3	24

6. State and prove the Lagrangian sufficiency theorem.

Let f and g be function given by

$$\begin{aligned} f(x,y,z) &= 3x + 2y - z \\ g(x,y,z) &= x^2 + y^2 + (z-5)^2 - 18 \end{aligned}$$

Show that f has a maximum and a minimum value on the set of points (x, y, z) in $\mathbb{R}^3$ which satisfy $g(x, y, z) = 0$. Use a Lagrangian multiplier to find the maximum and the minimum values and the points at which they are attained.