UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics M255: Linear Programming and Optimization

COURSE CODE	:	**MATHM255**
UNIT VALUE	:	**0.50**
DATE	:	**24-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

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TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Explain how to apply the simplex algorithm to find a basic non-negative solution x of a matrix equation $Ax = b$, if one exists.

   Apply the method to find a non-negative solution of the following equations:

$$\begin{array}{rcrcrcrcl} \xi_1 & & & - & \xi_3 & + & 4\xi_4 & = & 3 \\ 2\xi_1 & - & \xi_2 & & & & & = & 3 \\ 3\xi_1 & - & 2\xi_2 & & & - & \xi_4 & = & 1 \end{array}$$

2. Let (P) be the non-degenerate canonical maximum linear program

$$\text{maximize } c^{\mathsf{T}}x,$$
$$\text{subject to } x \geqq o \text{ and } Ax = b,$$

   where $A = (\alpha_{ij})$ is an $m \times n$ matrix, $b = (\beta_1, \ldots, \beta_m)^{\mathsf{T}} \in \mathbb{R}^m$ and $c = (\gamma_1, \ldots, \gamma_n)^{\mathsf{T}} \in \mathbb{R}^n$. Write down the dual program (P*). If x is feasible for (P) and y is feasible for (P*), show that

$$c^{\mathsf{T}}x \leqslant b^{\mathsf{T}}y.$$

   Prove that (P) and (P*) have the same optimal value. (Any assumptions about the simplex algorithm should be clearly stated, but need not be proved.)

   A tableau for a canonical maximum linear program (P) is

0	0	2	1	0	-1	1
1	0	4	0	0	-3	5
0	0	-2	0	1	1	4
0	1	-1	0	0	-2	2
0	0	2	0	0	-3	7

   Show that the objective function of (P) is unbounded above.

3. Define the terms *two-person zero-sum game* and *pure strategy* (for the two players R and C), and *mixed strategy* and *expected loss* (for C).

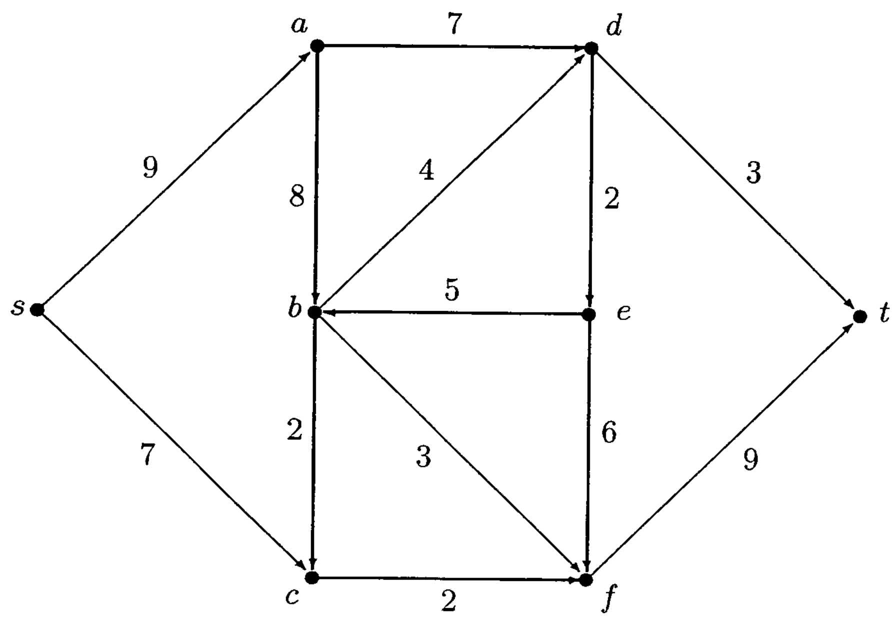
Without turning it into the associated linear program, describe the optimization problem which C must solve for an optimal strategy t.

Find optimal mixed strategies for R and C, and the value of the game, whose payoff matrix is

$$\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}.$$

4. Define the terms *network, capacity, flow, value* (of a flow), *cut* and *capacity of a cut.*

Find a maximum flow and a minimum cut for the following network:



5. Define the *optimal assignment problem*, and solve it when the assignment matrix is

$$\begin{bmatrix} 3 & 9 & 4 & 1 \\ 4 & 9 & 3 & 3 \\ 7 & 3 & 8 & 5 \\ 3 & 5 & 7 & 4 \end{bmatrix}.$$

Outline the method which is employed in the solution.

6. State and prove a necessary and sufficient condition for a digraph G to have a directed Euler circuit. (Any terms used should be carefully defined.)

   Show how to find a cyclic (0,1)-sequence which contains every triple of zeros and ones exactly once.

END OF PAPER