UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *B.Sc.(Econ)* *Coll Dip* *M.Sci.*

Mathematics M255: Linear Programming and Optimization

COURSE CODE	:	**MATHM255**
UNIT VALUE	:	**0.50**
DATE	:	**09-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**2 hours**

02-C0955-3-140

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Consider the following standard linear program, P.

$$\begin{array}{lrcrcrcr} \text{Maximise} & x & + & 2y & + & 5z & & \\ \\ \text{subject to} & -x & + & -y & + & z & \leqslant & -4 \\ & x & + & 2y & + & 3z & \leqslant & 24 \\ & x & , & y & , & z & \geqslant & 0 \end{array}$$

Write down an auxiliary program which could be used to find a basic feasible solution to P and explain the rationale behind your choice of objective function.

Use the two-phase simplex algorithm to find the optimal solution to P and give the value of the objective function at this optimal point.

After each phase, identify the basic feasible solution produced and check that it *is* feasible.

2. Let P be the standard linear program

$$\begin{array}{ll} \text{Maximise} & c^T.x \\ \\ \text{subject to} & Ax \leqslant b \ , \ x \geqslant 0 \end{array}$$

where x and c are vectors in $\mathbf{R}^k$, A is an $m \times k$ matrix and b is a vector in $\mathbf{R}^m$.

Write down the dual program to P.

Explain what it means for an array of numbers to be a valid simplex tableau for P.

What condition will be satisfied by the final simplex tableau for P.

State and prove the strong duality theorem for P and its dual.

3. Explain what is meant by a two-person zero-sum game with payoff matrix A.

   What is meant by an equilibrium for such a game?

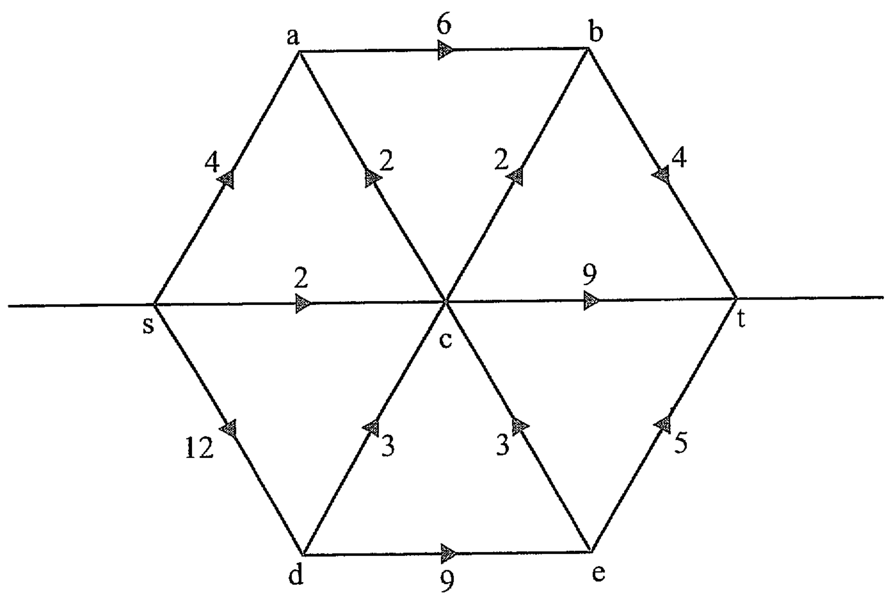
   Use the strong duality theorem to show that for any such game, there is an equilibrium pair of randomised strategies for the two players.

   Find the equilibrium for the game with payoff matrix

$$\begin{pmatrix} 4 & 7 & 6 \\ 6 & 2 & 3 \end{pmatrix}.$$

4. Prove that in a network with capacities, there is a cut whose value is equal to that of the maximum flow.

   Find a maximum flow and a minimum cut for the following network.



5. Let f and g be the functions given by

$$\begin{aligned} f(x,y,z) &= x^2+y^2+(z-2)^2 \\ g(x,y,z) &= xy-z. \end{aligned}$$

Show that f has a minimum value on the set of points (x,y,z) in $\mathbf{R}^3$ which satisfy

$$g(x,y,z)=0.$$

Use a Lagrange multiplier to find possible candidates for the minimum and determine which ones yield the lowest value for f: call this value J.

For the appropriate λ express $f(x,y,z)-\lambda g(x,y,z)$ in the form

$$\mathrm{J}+h(x,y,z)$$

where h is a sum of squares.

6. Suppose that sources $S_1, S_2, \ldots, S_m$ can supply respectively $s_1, s_2, \ldots, s_m$ units of a certain product and n destinations $D_1, D_2, \ldots, D_n$ demand $d_1, d_2, \ldots, d_n$ units respectively, where

$$\sum s_i = \sum d_j.$$

Given a cost matrix C, whose ij^{th} entry c_{ij} is the cost per unit of transporting from source S_i to destination D_j, explain with justification how to test whether a particular transportation arrangement is optimal.

Find an optimal transportation for the array

	D_1	D_2	D_3	D_4	
S_1	2	2	6	6	7
S_2	4	5	9	10	6
S_3	4	5	7	8	7
	2	7	5	6	20