UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sc.* *M.Sci.*

Mathematics C395: Graph Theory and Combinatorics

COURSE CODE : **MATHC395**

UNIT VALUE : **0.50**

DATE : **16-MAY-06**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

06-C1026-3-130

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Give the definition of a connected component of a graph. Assume that the graph $G(V, E)$ contains no cycle. Show that the number of connected components of G is $|V| - |E|$.
   (b) Assume T is a tree with vertex set $[n]$, $n \geqslant 3$. Define the Prüfer code of T. Find the tree whose Prüfer code is $(7, 3, 3, 1, 7, 3)$.
   (c) Give the definition of a bipartite graph. Show that a graph is bipartite if and only if it contains no odd cycle.

2. (a) State and prove the necessary and sufficient condition for the existence of an Euler circuit in a graph.
   (b) Assume G is a graph on $n \geqslant 3$ vertices with $\delta(G) \geqslant n/2$. Show that G contains a Hamilton cycle.
   (c) Give an example of a non-planar graph that has no subgraph isomorphic to K_5 or $K_{3,3}$.

3. (a) State the König-Hall theorem and use it to prove Hall's theorem on distinct representatives.
   (b) Give the definition of the chromatic number of a graph. Show that the chromatic number of a planar graph is at most 5.
   (c) State Euler's formula for planar graphs. Prove that $K_{3,3}$ is not planar.

4. (a) Define the Turán graph $T_r(n)$. State Turán's theorem. Prove that among all r-partite graphs on n vertices, the Turán graph has the largest number of edges.
   (b) State and prove Mantel's theorem.
   (c) Show that in every Red Blue colouring of the edges of K_n there is a monochromatic spanning tree.

5. (a) Define the k-tuple Ramsey number $R^k(s,t)$ and prove that it is finite, assuming the finiteness of the usual Ramsey numbers $R(s,t)$.

   (b) Give the definition of an antichain. Assume $\mathcal{A}$ is an antichain on ground set X. Is the set $\mathcal{A}^* = \{X \setminus A \ : A \in \mathcal{A}\}$ an antichain? Justify your answer. When $\mathcal{P}([9])$ is decomposed into symmetric chains, how many chains are there of size 10, of size 8?

   (c) State and prove the Erdős-Ko-Rado theorem.