UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics C395: Graph Theory and Combinatorics

COURSE CODE	:	**MATHC395**
UNIT VALUE	:	**0.50**
DATE	:	**05-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Give the definition of a tree. Show that every connected graph contains a spanning tree.
   (b) Find the tree whose Prüfer code is $(2, 1, 5, 5, 1, 2, 3)$.
   (c) Give the definition of an Euler circuit, and state the theorem on the existence of an Euler circuit in a graph.

2. (a) For which m and n does the complete bipartite graph $K_{m,n}$ contain a Hamilton cyle? Justify your answer.
   (b) Assume $n \geqslant 4$ is even. Construct a graph G on n vertices with $\delta(G) = (n-2)/2$ that contains no Hamilton cycle.
   (c) Give the definition of the chromatic number of a graph G. Show that every graph on n vertices and chromatic number k contains an independent set of size n/k.

3. (a) State the König-Hall theorem and use it to show that an r-regular ($r \geqslant 1$) bipartite graph G with bipartition classes X and Y has a complete matching from X to Y.
   (b) Decide whether $(1, 1, 1, 2, 2, 3, 4, 5, 5)$ is the degree sequence of a graph. In case it is, make a drawing of such a graph.
   (c) State Euler's formula for planar graphs. Prove that K_5 is not planar.

4. (a) Define the Turán graph $T_r(n)$. State Turán's theorem.
   (b) Prove that among all r-partite graphs on n vertices, the Turán graph has the largest number of edges.
   (c) State and prove the local LYM inequality. When does equality hold?

5. (a) Define the Ramsey numbers $R(s, t)$. Show that $R(3, 3) = 6$
   (b) When $\mathcal{P}([8])$ is decomposed into symmetric chains, how many chains are there? How many chains are there of size 9, of size 8, and of size 1?
   (c) State and prove Fisher's inequality.

MATHC395 END OF PAPER