UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C395: Graph Theory and Combinatorics

COURSE CODE	:	**MATHC395**
UNIT VALUE	:	**0.50**
DATE	:	**17-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Prove that if G is a graph with $n \geqslant 3$ vertices and $\delta(G) \geqslant n/2$ then G contains a Hamilton cycle.

   (b) Define the Turán graph $T_r(n)$.

   (c) State Turán's Theorem.

   (d) Prove that if G is a graph with n vertices and $\delta(G) > n/2$ then G contains a copy of K_3.

2. (a) Define the chromatic number $\chi(G)$ of a graph G.

   (b) Show that if G contains a complete subgraph on k vertices then $\chi(G) \geqslant k$.

   (c) Show that if $\chi(G) = t$ then G contains an independent set of size at least n/t.

   (d) Let G be a graph with vertex set V, and let $\overline{G}$ be the complementary graph: so $V(\overline{G}) = V$ and $E(\overline{G}) = V^{(2)} \setminus E(G)$. Prove that $\chi(G)\chi(\overline{G}) \geqslant |V|$.

   (e) Prove that if G is a graph then $\chi(G) \leqslant \Delta(G) + 1$.

3. (a) Define a symmetric chain in $\mathcal{P}([n])$.

   (b) Prove that, for every $n \geqslant 1$, $\mathcal{P}([n])$ can be decomposed into symmetric chains.

   (c) How many chains are there in a symmetric chain decomposition of $\mathcal{P}([5])$? How many of each length?

   (d) State and prove Sperner's Lemma.

4. (a) Define the lexicographic and colex orders on $[n]^{(r)}$.

   (b) Write down the elements of $[4]^{(2)}$ in colex order.

   (c) State the Kruskal-Katona Theorem, taking care to define all the terms.

   (d) State and prove the Erdős-Ko-Rado Theorem on intersecting hypergraphs.

5. (a) Define the Ramsey numbers $R(s,t)$ and prove that they exist for all $s, t \geqslant 2$.

   (b) Prove that $R(3,3) = 6$.

   (c) Prove that if the edges of K_{17} are coloured using three colours then there is a monochromatic triangle.

MATHC395 END OF PAPER