UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C395: Graph Theory and Combinatorics

COURSE CODE	:	**MATHC395**
UNIT VALUE	:	**0.50**
DATE	:	**20-MAY-03**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

03-C0932-3-80

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Give the definition of a tree. Show that every tree with at least two vertices has at least two leaves.
   (b) For which m and n is the complete bipartite graph $K_{m,n}$ planar?
   (c) Decide whether $(1, 1, 1, 2, 2, 3, 4, 5, 5)$ is the degree sequence of a graph. In case it is, make a drawing of such a graph.

2. (a) Give the definition of an Euler circuit, and state the theorem on the existence of an Euler circuit in a graph.
   (b) Assume $n \geqslant 4$ is even. Construct a graph G on n vertices with $\delta(G) = (n-2)/2$ that contains no Hamilton cycle.
   (c) Give the definition of the chromatic number, $\chi(G)$, of a graph G. Show that then $\chi(G) \leqslant \triangle(G) + 1$

3. (a) State the König-Hall theorem and use it to show that an r-regular ($r \geqslant 1$) bipartite graph G with bipartition classes X and Y has a complete matching from X to Y.
   (b) Construct a decomposition of the edge set of K_9 into edge-disjoint Hamilton cycles.
   (c) State Euler's formula for planar graphs. Prove that K_5 is not planar.

4. (a) Define the Turán graph $T_r(n)$. State Turán's theorem.
   (b) State and prove the LYM inequality.
   (c) When $\mathcal{P}([8])$ is decomposed into symmetric chains, how many chains are there? How many chains are there of size 9, of size 8, and of size 7?

5. (a) Define the Ramsey numbers $R(s, t)$. Show that $R(s, s) \geqslant 2^{s/2}$ if $s \geqslant 3$.
   (b) Show that in every 3-colouring of the edges of K_{17} there is a monochromatic triangle.
   (c) Give the definition of an antichain. State the strong form of Sperner's theorem.