UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Mathematics C395: Graph Theory and Combinatorics

COURSE CODE	:	**MATHC395**
UNIT VALUE	:	**0.50**
DATE	:	**22-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0927-3-80

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Show that the edge set of K_n can be decomposed into pairwise edge-disjoint Hamilton paths if and only if n is even.
   (b) Show that a graph G is a tree if and only if it is connected and deleting any edge $e \in E(G)$ makes the graph disconnected.
   (c) Is $(1, 1, 1, 1, 2, 2, 2, 4)$ the degree sequence of a tree?

2. (a) Prove that if $G = (V, E)$ is a graph on n vertices, $n \geq 3$ and $d(v) \geq n/2$ for every $v \in V$, then G contains a Hamilton cycle.
   (b) Show that the Petersen graph has cycles of length 5,6,8,9.
   (c) Give the definition of the chromatic number, $\chi(G)$, of a graph G. Show that then $\chi(G) \leq \triangle(G) + 1$

3. (a) State and prove Hall's theorem on the system of distinct representatives.
   (b) Prove that among all r-partite graphs on n vertices, the Turán graph has the largest number of edges.
   (c) State Euler's formula for planar graphs. Prove that K_5 is not planar.

4. (a) State and prove Sperner's lemma (without specifying the case of equality).
   (b) State and prove the Local LYM inequality. When does equality hold?
   (c) When $\mathcal{P}([8])$ is decomposed into symmetric chains, how many chains are there? How many chains are there of size 9, of size 8, and of size 7?

5. (a) Define the Ramsey numbers $R(s, t)$, and prove that they exist.
   (b) Assume that all pairs of positive integers are coloured Red, Blue or Green. Prove that there is an infinite subset of the positive integers all of whose pairs are coloured with the same colour.
   (c) For which n and m is the complete bipartite graph $K_{m,n}$ planar?

MATHC395 END OF PAPER