UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A.   *B.Sc.*

Mathematics A1B: Elementary Mathematics 2

COURSE CODE   : **MATHA01B**

UNIT VALUE   : **0.50**

DATE   : **23–MAY–06**

TIME   : **14.30**

TIME ALLOWED   : **2 Hours**

TURN OVER

All *questions may be answered but only marks obtained on the best* **five** *questions will count. The use of an electronic calculator* **is** *permitted in this examination.*

1. Find the following:

    (i) $\int \frac{1}{1+x^2} dx$,

    (ii) $\int \log x \, dx$,

    (iii) $\int \frac{x}{x^2-1} dx$.

2. a) Sketch the graph of $\tan x$. Explain how the inverse $\tan^{-1} x$ is defined and determine the derivative of $\tan^{-1} x$.

   b) Determine whether the improper integrals (i) $\int_0^1 \log x \, dx$ and (ii) $\int_1^\infty \frac{1}{x^3} dx$ converge and, when convergent, determine the value.

3. a) Find the area under the curve $y = x^2(1-x)^2$ between $x = 0$ and $x = 1$.

   b) Consider the curve $y = x(1-x)$ between $x = 0$ and $x = 1$. Find the volume of the body of revolution formed by rotating this curve about the x-axis.

4. By forming partial fractions, or otherwise, evaluate

    (i) $\int \frac{4x+3}{x^2+5x+6} dx$,

    (ii) $\int \frac{2x+1}{(x+2)^2} dx$.

5. (i) A particle at the origin is acted on by 3 forces: a force of 10 in the direction of $(0,2)$, a force of 13 in the direction of $(5, 12)$, and a force of 5 in direction $(-1,0)$. In what direction does the particle begin to move? What is the magnitude of the resultant force?

(ii) Find a vector which is perpendicular to $\mathbf{u} = (1,2,3)$ and $\mathbf{v} = (-1,1,0)$.

(iii) Find the equation of the plane through the points $(1, 0, -1), (2, 2, 2)$ and $(0, 1, -1)$.

6. (i) An item in a shop originally cost £100. The price is increased by 15%; then the item is put in a sale in which all items are reduced by 15%. How much does it cost in the sale?

(ii) If $f = \dfrac{x^2 y}{z^3}$ and x is accurate to $\pm 4\%$, y is accurate to $\pm 1\%$, z is accurate to $\pm 0.2\%$, what is (approximately) the accuracy of f?

7. (i) Solve the differential equation

$$\frac{dy}{dx} = y \log(x + 2)$$

where $y = 27e^{-1}$ when $x = 1$.

(ii) Solve the differential equation

$$\frac{d^2 y}{dx^2} + 25y = 0$$

given that $y = 1$ and $\dfrac{dy}{dx} = 3$ when $x = 0$. Find the amplitude of y.

MATHA1B   END OF PAPER