

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.A. B.Eng. B.Sc. M.Sci.

Mathematics A1B: Elementary Mathematics 2

COURSE CODE : MATHA01B

UNIT VALUE : 0.50

DATE : 25–MAY–05

TIME : 14.30

TIME ALLOWED : 2 Hours

All questions may be attempted but only marks obtained on the best five solutions will count.

The use of an electronic calculator is permitted in this examination.

1. Find the following

(i) $\int \frac{x^2}{1+x^3} dx$

(ii) $\int x \cos x dx$

(iii) $\int x^{-2/3} \ln(x^{1/3}) dx$.

2. Find the stationary values of the following functions and determine their nature

(i) $y = 3x^4 - 8x^3 + 6x^2$

(ii) $y = x^2 e^x$.

3. (i) Find the area under the curve $y = (1 - x^2)^{-1/2}$ between $x = 0$ and $x = \frac{1}{2}$.

(ii) Find the volume of the solid of revolution obtained by rotating the curve $y = x(1 - x)$ between $x = 0$ and $x = 1$ about the x -axis.

4. Define $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$.

(i) Show that $\frac{d}{dx} \cosh x = \sinh x$, $\frac{d}{dx} \sinh x = \cosh x$.

(ii) $\cosh^2 x - \sinh^2 x = 1$.

(iii) Find $\int (x^2 - 1)^{-\frac{1}{2}} dx$.

5. (a) Let $u = (0, 1, 1)$, $v = (0, 0, 1)$. Find the magnitude of u and v and find the angle between u and v .

(b) A body at the origin is acted on by a force of magnitude 2 in the direction of the vector $(-1, 1, 0)$ and a force of magnitude 1 in the direction of the vector $(1, 1, 0)$. Find the direction in which the body starts to move.

(c) Find the equation of the plane through $(1, 1, 1)$, $(3, -2, 2)$, $(-1, -1, 5)$.

6. (a) Let $y = e^{-x^2}$. Find y'' and show that $|y''(x)| \leq 2$ for all x between 0 and 1.
- (b) Use the Trapezium rule to find an approximate value for the integral $\int_0^1 e^{-x^2} dx$ with an error less than 0.01. [You may leave your answer in powers of e and you may assume that the error in the Trapezium formula is $\leq nd^3K/12$ where K is the maximum value of $|y''|$ in the given range, $d = \frac{1}{n}$ where n is the number of points used.]
7. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y-1}{1+x^2}$$

given that $y = 4$ when $x = 1$.

- (b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 16y = 0$$

given that $y = 1$ and $\frac{dy}{dx} = 4$ when $x = 0$.

Find the amplitude of y .