

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.A. B.Eng. B.Sc.

Mathematics A1B: Elementary Mathematics 2

COURSE CODE : **MATHA01B**

UNIT VALUE : **0.50**

DATE : **18–MAY–04**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

All questions may be attempted but only marks obtained on the best **five** solutions will count.

The use of an electronic calculator is permitted in this examination.

1. Find the stationary points of the function $y = x^3/3 + x^2/2 - 2x + 1$ and describe their nature. What is the gradient of the tangent to the graph of this function when it cuts the y -axis? Sketch the curve.

2. (a) Define the function $\sin^{-1} x$, sketch its graph and find its derivative with respect to x .
(b) Find $\int \sin^{-1} x \, dx$.
(c) Find $\int \log x \, dx$.

3. Find the following:
 - (a) $\int \frac{x^3}{x^4 + 42} \, dx$,
 - (b) $\int x e^x \, dx$,
 - (c) $\int x^2 \cos x \, dx$,

4. (a) Using the formula for calculating a solid of revolution, find the volume of a sphere of radius r .
(b) Show that the improper integral $\int_0^\infty \sin x e^{-x} \, dx$ is convergent and find its value.

5. Let $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (2, 3, -4)$.

- (a) Find $\mathbf{u} \cdot \mathbf{v}$ (the scalar product).
- (b) Find $\mathbf{u} \times \mathbf{v}$ (the vector product).
- (c) If θ is the angle between \mathbf{u} and \mathbf{v} , find $\cos \theta$.

6. (a) Use the Trapezium rule with 5 divisions to find an approximate value for the integral $\int_0^1 x^2 dx$. Find the true value of the integral and hence find the error of the approximate answer.

(b) Suppose a cubical box of side x is measured to an accuracy of $r\%$, where r is a small number. What percentage error would you expect when you calculate its volume?

7. Solve the following differential equations.

(a) $\cos^2 x \frac{dy}{dx} = y^2$.

(b) $\frac{dy}{dx} = \frac{e^{-x}}{y}$.