

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. B.Eng. M.Sci.

Mathematics A1B: Elementary Mathematics 2

COURSE CODE : MATHA01B

UNIT VALUE : 0.50

DATE : 28-MAY-03

TIME : 14.30

TIME ALLOWED : 2 Hours

All questions may be attempted but only marks obtained on the best five solutions will count.

The use of an electronic calculator is permitted in this examination.

1. (a) Find the stationary points of the function $y = x^2e^{-x}$ and determine their nature.
(b) Sketch the graph of $\sin x$. Explain how $\sin^{-1}(x)$ ($= \arcsin(x)$) is defined, and determine the derivative of $\sin^{-1}(x)$ from your definition.
2. Find the following:
 - (i) $\int \frac{x}{x^2+1} dx$,
 - (ii) $\int x \sin(2x) dx$,
 - (iii) $\int \frac{1}{x}(\log x)^2 dx$.
3. (a) Find the volume of the solid of rotation obtained by rotating the curve $y = \sqrt{x(1-x)}$ between $x = 0$ and $x = 1$ about the x -axis.
(b) Find mean and root mean square of $\sin x$ between $x = 0$ and $x = 2\pi$.
4. Find $\int \sin^{-1} x dx$.
5. (a) Let $\mathbf{u} = (1,1,0)$, $\mathbf{v} = (1,2,1)$. Find the magnitude of \mathbf{u} and of \mathbf{v} , and find the angle between \mathbf{u} and \mathbf{v} .
(b) A body at the origin is acted on by a force of magnitude 3 in the direction of the point $(1,1,0)$, and a force of magnitude 5 in the direction of the point $(0,3,4)$. Find the unit vector in the direction in which the body starts to move.
(c) Find the equation of the plane through the points $(1,2,2)$, $(4,0,1)$ and $(-1,1,5)$.

6. (a) The quantities p and x are known to be related by an equation of the form $p = ax^n$. By drawing a suitable graph, find approximate values of a and n , if experimental values of p and x are as follows:

x	1	2	3	4	5
p	1.8	8.2	17	30	55

- (b) If $f = a^2b^{-3}c^{1/2}$, and the values of a , b and c are correct to an accuracy of $\pm 0.5\%$, $\pm 0.2\%$ and 0.1% (respectively), what is, approximately, the corresponding percentage accuracy of f ?
- (c) Give a better approximation than 1 for $(1 + 10^{-20})^{-3}$.
7. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{(y^2 - 1)}{(x^2 - 1)},$$

given that $y = 2$ when $x = 2$.

- (b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 16y = 0,$$

given that $y = 1$ and $\frac{dy}{dx} = 4$ when $x = 0$. Find the amplitude of y .