UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.A.   B.Eng.   B.Sc.   M.Sci.

Mathematics A1B: Elementary Mathematics 2

COURSE CODE   : **MATHA01B**

UNIT VALUE   : **0.50**

DATE   : **14-MAY-02**

TIME   : **14.30**

TIME ALLOWED   : **2 hours**

TURN OVER

*All questions may be attempted but only marks obtained on the best **five** solutions will count.*

The use of an electronic calculator is permitted in this examination.

1. Find the following:

   (i) $\int x \cos x^2 \, dx$,

   (ii) $\int x^2 \log x \, dx$,

   (iii) $\int \cos x \sin^4 x \, dx$.

2. (a) Sketch the graph of $\cos x$. Explain how $\cos^{-1}(x)$ ($= \arccos(x)$) is defined, and determine the derivative of $\cos^{-1}(x)$ from your definition.

   (b) Determine whether improper integrals (i) $\int_0^1 x^{-2/3} \, dx$ and (ii) $\int_0^1 x^{-4/3} \, dx$ converge, and, when convergent, determine the value.

3. (a) Find the area under the curve $y = x(1 - x^2)^3$ between $x = 0$ and $x = 1$.

   (b) Show how to find the volume of the sphere of radius 1 by treating it as a solid of revolution.

4. Find $\int (1 + x^2)^{-3/2} \, dx$.

5. (a) Let $\mathbf{u} = (1,2,0)$, $\mathbf{v} = (0,2,-1)$. Find the magnitude of $\mathbf{u}$ and of $\mathbf{v}$, and find the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (b) A body at the origin is acted on by a force of magnitude 2 in the direction of the point $(1,1,1)$, and a force of magnitude 3 in the direction of the point $(2,-1,-1)$. Find the unit vector in the direction in which the body starts to move.

   (c) Find the equation of the plane through the points $(1,0,2)$, $(1,1,1)$ and $(0,1,-2)$.

6. (a) Let $y = \sin(x^2)$. Find y'' and show that $|y''(x)| \leq 6$ for all x between 0 and 1.

   (b) Use the Trapezium rule to find an approximate value for the integral $\int_0^1 \sin(x^2) \, dx$ with an error less than 0.01.

   [You may assume that the error in the Trapezium formula for $\int_a^b y \, dx$ is $\leq nd^3 K/12$, where K is the maximum value of $|y''|$ in the given range, n is the number of divisions, and $d = (b - a)/n.]

7. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{x(y^2 + 1)}{y(x^2 - 1)},$$

given that $y = 2$ when $x = 2$.

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = 0,$$

given that $y = 2$ and $\frac{dy}{dx} = 2$ when $x = 0$. Find the amplitude of y.