UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.*

Mathematics A1A: Elementary Mathematics 1

COURSE CODE	:	**MATHA01A**
UNIT VALUE	:	**0.50**
DATE	:	**19-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

06-C0993-3-30

TURN OVER

All questions may be answered, but only marks obtained on the best five solutions will count.
The use of an electronic calculator is permitted in this examination.

1. Simplify the following expressions:

(a) $\left(\frac{a^n}{b^n}\right)\left(\frac{b^{n-1}}{a^{n-1}}\right)$ where $a \neq 0$ and $b \neq 0$

(b) $$\frac{\frac{1}{x}-\frac{3}{x^2}+\frac{7}{x^3}}{\frac{-4}{x}-\frac{3x-2}{x^2}+\frac{3x^2-5x+2}{x^3}}$$

(c) $$\left(\left(\sqrt[3]{x}\right)^2 \frac{y^5}{\sqrt[3]{z}}\right)\frac{\sqrt[3]{x}}{\left(x \cdot z^{\frac{1}{3}} y\right)^2}$$

2. (a) Find the equations of the lines passing through the following pairs of points:
   - $\{(3,1), (2,4)\}$
   - $\{(2,0), (5,7)\}$

   (b) Determine whether these lines are parallel, orthogonal or neither.
   (c) Find the equation of the perpendicular line to each of the two lines defined in (a), through the point (1,1).
   (d) Graph the four lines.

3. By using the formulas for the summation and subtraction of angles, show that:
   (a) $2\cos\theta\cos\phi = \cos(\theta+\varphi)+\cos(\theta-\varphi)$
   (b) $\sin(\frac{5\pi}{2}-\theta)+\cos(3\pi+\theta)=0$
   (c) Find exact values for the following (without using a calculator):
   - $\cos(12\pi)$
   - $\sin(\frac{2\pi}{3})$
   - $\tan(\frac{9\pi}{4})$

4. (a) Find where the curve $20x^2-5y+40x+15=0$ meets the y-axis.
   (b) Find the real solutions (if any) of the following equations.
   - $x^2-3x+1=0$
   - $z^2-6z=-12$
   - $2w(w+3)=(w-1)^2$

5. Differentiate the following expressions with respect to x:

(a) $\log(\cos(x))$

(b) $\tan(\sin(x^2))$

(c) $e^{x(x^2-3)}$

(d) $(x^2-2x+1)^{-\frac{3}{4}}$

6.

(a) Write the following expressions in the form $z = a + bi$ where a, b are real numbers. If $z_1 = 3+3i$ and $z_2 = 3-4i$.

- $z_1^* z_2$
- $\dfrac{z_1}{z_2}$

Note: z_1^* is the "complex conjugate" of z_1.

(b) State De Moivre's Theorem and use it to compute:

z_1^{12}, z_1^5, z_2^3, and z_2^8 where

$$z_1 = 1+i \quad \text{and} \quad z_2 = \sqrt{5}\left(\cos\frac{\pi}{24} + i\sin\frac{\pi}{24}\right).$$

7.

(a) State the Binomial Theorem and use it to find the coefficient of x^8 in the expansion of $(1+x)^{23}$. Write the coefficient as a product of prime factors.

(b) Write down Maclaurin's formula for expanding a function $f(x)$ as a series in x and use it to find a series for $(1+x)^{\frac{3}{4}}$ up to and including the x^3 term.