UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Eng.* *B.Sc.* *M.Sci.*

Mathematics A1A: Elementary Mathematics 1

COURSE CODE : **MATHA01A**

UNIT VALUE : **0.50**

DATE : **17-MAY-05**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

05-C0908-3-50

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator **is** *permitted in this examination.*

1. Simplify the following:

   (a) $\dfrac{(x-3y)(x^2-4xy)(xy-y^2)(x+2y)}{x(yx^2-6y^3-xy^2)}$,

   (b) $(x+y)^3-(x-y)^3+y(x+y)^2-x(x+y)^2$,

   (c) $(x^{\frac{3}{2}}y\sqrt{z})^3\dfrac{y^5}{\sqrt[4]{z^{12}}}$.

2. Sketch the curve $y=x^2-3x+2$ giving the coordinates of the points where it crosses the x-axis.

   The line L_1 passes through the points $(2,-2)$ and $(3,-4)$. Find the equation of the line L_1 and the coordinates of the points at which it cuts the curve.

   A second line L_2 is perpendicular to L_1 and passes through the point $(1,1)$. Find the equation of the line L_2 and add the lines L_1, L_2 to your sketch. [There is no need to find the coordinates of the points where L_2 cuts the curve.]

3. Write down the formulae for $\sin(A+B)$ and $\sin(A-B)$ in terms of $\sin A, \sin B, \cos A$, and $\cos B$. Apply one formula to

   $$\sin(\frac{\pi}{3}+\frac{\pi}{4})$$

   to deduce that

   $$\sin\frac{7\pi}{12}=\frac{1+\sqrt{3}}{2\sqrt{2}}.$$

   Find a similar result for $\sin\frac{\pi}{12}$.

   Write down the exact values of $\sin\frac{5\pi}{12}$, $\sin\frac{13\pi}{12}$ and $\cos\frac{\pi}{12}$.

4. Differentiate the following with respect to x:

(a) $\dfrac{x^2+1}{x+1}$,

(b) $x^{\frac{1}{2}}e^{x^2}$,

(c) $\dfrac{1}{x^3} + \ln(3x^3)$,

(d) $\sin(x \sin x)$.

5. (a) Two circles, centres A and B and each of radius 5cm.,have their centres 8cm.apart and intersect along the line PQ. Find the length PQ. Denote the angle PAB by α and show that $\cos\alpha = 4/5$.

Assuming the formula for the area of a circle, show that the area of the sector PAQ of the circle centre A is $(25\alpha)\text{cm}^2$. Find, as a function of α, the area common to the two circles.

(b) Sketch the curve $y = 1/x^2$ and find the equation of the tangent where $x = 1$.

6. Write down the formula for the MacLaurin expansion of $f(x)$ in powers of x. Apply it to the function $(1+x)^n$, where n is a positive integer, to show that

$$(1+x)^n = \sum_{r=0}^{n} \frac{n!}{r!(n-r)!} x^r.$$

Find

(i) the coefficient of x^8 in $(1+x^2)^{14}$,

(ii) the coefficient of x^{-7} in $\left(2 - \dfrac{3}{x}\right)^{10}$.

7. (a) Show that the equation $z^2 + 2z + 3 = 0$ has no real roots.

Find the roots z_1, z_2 and show that they are complex conjugates.

Show that $z_1 + z_2 = -2$ and that $z_1 z_2 = 3$.

Choose either root to be z_1 and write z_1/z_2 in the form $a + ib$ where a, b are real numbers.

(b) Write $1 + \mathrm{i}$ in the form $re^{\mathrm{i}\theta}$ and hence write $(1+\mathrm{i})^{23}$ in the form $c + \mathrm{i}d$ where c, d are real numbers.

END OF PAPER