

**UNIVERSITY COLLEGE LONDON**

University of London

**EXAMINATION FOR INTERNAL STUDENTS**

For The Following Qualifications:–

*B.A.*    *B.Eng.*    *B.Sc.*

**Mathematics A1A: Elementary Mathematics 1**

COURSE CODE            :   **MATHA01A**

UNIT VALUE             :   **0.50**

DATE                     :   **13-MAY-04**

TIME                     :   **14.30**

TIME ALLOWED         :   **2 Hours**

All questions may be attempted but only marks obtained on the best **five** solutions will count.

The use of an electronic calculator is permitted in this examination.

1. Simplify the following expressions.

(a) 
$$\frac{(x-y)(x^3-4y)(x+y)(xy-1)}{(x+5y)(x^2-y^2)}$$

(b) 
$$\left( (\sqrt[3]{x})^2 \frac{y^5}{\sqrt[3]{z}} \right) \frac{\sqrt[3]{x}}{(xz^{1/3}y)^2}$$

(c) 
$$x(x-y+xy)^2 - (x-y)^3 - y(y-x)^2$$

2. (a) Find the points where the curve  $y^2 + 20x + 4y - 60 = 0$  meets the  $y$ -axis.

(b) Write the equation of the straight line passing through the points  $(2, 3)$  and  $(-1, 4)$ .

(c) Write the equation of the straight line passing through the point  $(2, -4)$  and perpendicular to the line  $5x + 3y - 8 = 0$ .

3. (a) Find the values of  $\sin \theta$  and  $\cos \theta$ , given  $\tan \theta = -3/4$ .

(b) By using the formulas for the summation and subtraction of angles, show that

(i)  $2 \cos \theta \cos \phi = \cos(\theta + \phi) + \cos(\theta - \phi)$ , and verify the identities

(ii)  $\sin 2\theta = 2 \sin \theta \cos \theta$  and (iii)  $1 = \sin^2 \theta + \cos^2 \theta$ .

(c) Find exact values of the following expressions:

(i)  $\cos(12\pi)$ ; (ii)  $\tan(\frac{17\pi}{4})$ ; (iii)  $\sin(\frac{5\pi}{2})$ .

4. Differentiate the following expressions with respect to  $x$ .

(a)  $(x^2 + 1)^{1/2}$ .

(b)  $e^x(x^2 - 2)$ .

(c)  $\log(8 + x^3)$ .

(d)  $\cos(\sin(x^2))$ .

5. (a) Write the following expressions in the form  $z = a + ib$ , where  $z$  is a complex number and  $a$  and  $b$  are real numbers:
- (i)  $z = z_1 z_2$ , where  $z_1 = -1 + i$ ,  $z_2 = \frac{1}{2} - i$ ;
- (ii)  $z = z_1^{12}$ , where  $z_1 = \sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$ .
- (b) State De Moivre's Theorem and use it to prove the double-angle formulas, that relate  $\sin 2\theta$  and  $\cos 2\theta$  to  $\sin \theta$  and  $\cos \theta$ .
6. (a) State the Binomial Theorem and use it to find the coefficient of  $x^4$  in  $(1 + x)^{16}$ .
- (b) Write down Maclaurin's formula for expanding a function  $f(x)$  as a power series in  $x$  and use it to find the power series for  $f(x) = (1 + x)^a$ ,  $a$  real, up to the term in  $x^n$ .
7. (a) Find the equation of the tangent and normal to the curve  $y = \frac{1}{x}$  at  $x = 2$ .
- (b) Find the critical value and determine the relative minimum and maximum values of the following functions:
- (i)  $f(x) = x^2 - 8x$ ;      (ii)  $f(x) = 2x^3 - 24x + 5$ .