UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.A. *B.Eng.* *B.Sc.* *M.Sci.*

Mathematics A1A: Elementary Mathematics 1

COURSE CODE	:	**MATHA01A**
UNIT VALUE	:	**0.50**
DATE	:	**16-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0895-3-50

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator **is** *permitted in this examination.*

1. Simplify the following expressions:

   (a) $c^2 - 2ab + (a+b-c)\,(a+b+c)$,

   (b) $\dfrac{\left(\frac{1}{\sqrt[5]{x}}\, y^6\, z^4\right)^3 z^{12}}{y^{18}\, x^3}$,

   (c) $\left(1 - x^{\frac{2}{3}}\right)\left(1 + x^{\frac{2}{3}} + x^{\frac{4}{3}} + x^2 + x^{\frac{8}{3}} + x^{\frac{10}{3}}\right)$.

2. (a) Write down the equation of the straight line through the points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$. What is the equation when $x_1 = x_2$? Find the equation of a straight line through the points $(4, 3)$ and $(6, 7)$ and find the shortest distance of this line to the origin.

   (b) Find the two points where the curves $y = 7x^2 - 24x + 10$ and $y = 8x^2 - 33x + 24$ meet and find the distance between these two points .

3. (a) Define the trigonometric functions $\sin\theta$, $\cos\theta$, $\tan\theta$ and $\cot\theta$ for a *general* angle θ. Sketch the graphs of $\cos\theta$ and $\sin\theta$ in the interval $-3\pi \leqslant \theta \leqslant 3\pi$.

   (b) By comparing the area of a certain sector of a circle with the area of two triangles, show that $\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{1}{2}\tan\theta$ for small values of $\theta > 0$. Complete the proof that $\dfrac{\sin\theta}{\theta} \to 1$ as $\theta \to 0$.

4. Differentiate the following with respect to x:

   (a) $(x^3 + x^2 + 5x + 3)(x + 3)$,

   (b) $\dfrac{x^3 + x^2 + 1}{x + \cos x}$,

   (c) $\sin(x^3)$,

   (d) $\exp(\sin(x^3))$.

5. Write the following expressions in the form $a + ib$ where a and b are real numbers

(a) $(3 + 6i)(3 + 2i)$, (b) $\frac{3+6i}{3+2i}$, (c) $e^{i\pi/4}$,
(d) $(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7})^{21}$.

6. (a) Write down Maclaurin's formula for expanding a function $f(x)$ as a power series in x and use it to find the power series for $f(x) = (1 + x)^{1/3}$, up to and including the x^3 term. Use the first two terms of the series to find an approximation to $\sqrt[3]{1012}$ to two places of decimals *without* using a calculator.

(b) Sketch the graphs of $f(x) = e^x$ and $g(x) = \log x$. Find the points on each curve where the gradient is 1 and show that the curves have a common normal at these points.

7. (a) If $f(x) = 2x^3 - 21x^2 + 60x + 10$ find the points for which $f'(x) = 0$. For which of these points does $f(x)$ have a local maximum?

(b) A rectangular plot of land is enclosed on three sides by fences whose total length is 100m. The fourth side is bounded by a wall. Find the maximum area that can be enclosed.