UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M234: Electricity and Magnetism

COURSE CODE : **MATHM234**

UNIT VALUE : **0.50**

DATE : **05-MAY-05**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

05-C0967-3-50

TURN OVER

*All questions may be attempted but only marks obtained on the best **four** solutions will count.*
*The use of an electronic calculator is **not** permitted in this examination.*

1. Consider the non-relativistic motion of a particle of mass m and charge q in an electric field $\mathbf{E} = (E, 0, 0)$ and magnetic flux density $\mathbf{B} = (0, B, 0)$, where E and B are constants. The particle starts at rest at the origin at time $t = 0$.
   (a) State the equation of motion, and show that the particle's path remains in a plane which should be identified.
   (b) Solve the equation of motion for the particle's velocity as a function of time.
   (c) Show that there is a time $T > 0$ at which the particle is again at rest, and find the value of the smallest such time.

2. (a) State and prove Coulomb's law in a vacuum for the electrostatic force between two charged point particles.
   (b) Using Cartesian coordinates (x, y, z), suppose that $x \geq 0$, $y \geq 0$ is vacuum and the rest of space is occupied by a grounded conductor. In the electostatic limit, what are the boundary conditions on the surface of the conductor? Find the electric field $\mathbf{E}$ everywhere for this system when a point charge q is placed at $(a, a, 0)$, with $a > 0$, and find the force on the charge. *Hint: use the method of images.*

3. (a) State the vacuum versions of Maxwell's equations and show that they imply conservation of charge.
   (b) In a simple conductor $\mathbf{J} = \sigma\mathbf{E}$, where $\mathbf{J}$ is the current density, $\mathbf{E}$ is the electric field and σ is the conductivity, which we assume is constant and uniform in the conductor. Show that any charge density inside the conductor decays exponentially in time at a rate which should be determined.
   (c) Define the field $\mathbf{K} = \mathbf{E} + \alpha\mathbf{B}$, where $\mathbf{E}$ and $\mathbf{B}$ have their usual meanings, and α is a (complex) constant. Show that, in a source-free vacuum, α can be chosen so that
   $$\nabla \times \mathbf{K} = \beta \frac{\partial \mathbf{K}}{\partial t},$$
   where β is another (complex) constant, and determine all the possible values of α and the corresponding values of β.

4. (a) State the electromagnetic media form of Maxwell's equations in differential form, giving the definitions of the fields $\mathbf{D}$ and $\mathbf{H}$. What are the physical interpretations of the polarization field $\mathbf{P}$ and magnetization field $\mathbf{M}$?

   (b) Consider a body occupying a region V with constant and uniform magnetization $\mathbf{M}_0$ in a vacuum. Show that the magnetic field $\mathbf{H}$ can be expressed as an integral over the surface S of V, and give its general form.

   (c) Using the result of 4b, find an approximation to the magnetic field $\mathbf{H}$ valid far from the *ends* of a long thin circular cylinder, with magnetization $\mathbf{M}_0$ parallel to the axis of the cylinder. The cylinder has radius r and length $2a$, with $a \gg r$.

5. (a) Starting from the vacuum versions of Maxwell's equations, state and prove the (standard version) of Poynting's theorem in a vacuum.

   (b) What is the physical interpretation of Poynting's theorem?

   (c) Verify Poynting's theorem for an electromagnetic plane wave in a vacuum, and show that the ratio of the time-averaged Poynting vector and the time-averaged energy density suggests that the energy moves at the speed of light.

6. A superconductor is a material that has no direct-current resistance, satisfies the vacuum version of Maxwell's equations and under steady-state conditions

$$\nabla \times \mathbf{J} = -\alpha \mathbf{B},$$

where $\mathbf{J}$ is the current density, $\mathbf{B}$ is the magnetic flux density and α is a material constant of the superconductor.

   (a) Show that $\mathbf{B}$ satisfies

$$\nabla^2 \mathbf{B} = \mathit{const.}\,\mathbf{B},$$

   and determine the constant.

   (b) Suppose that the superconductor occupies a half-space, which we take to be $x \geq 0$ in the Cartesian coordinates (x, y, z). Show that a consistent solution in $x \geq 0$ exists of the form $\mathbf{B} = (0, 0, B(x))$, with $B(0) = B_0$, and find the solution for $B(x)$ which decays as x tends to infinity.

   (c) Find the corresponding solution for the current density $\mathbf{J}$ in $x \geq 0$.