UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M234: Electricity and Magnetism

COURSE CODE	:	**MATHM234**
UNIT VALUE	:	**0.50**
DATE	:	**30-APR-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

04-C0961-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best ***four*** *solutions will count.*
The use of an electronic calculator is ***not*** *permitted in this examination.*

1. (a) State the vacuum versions of Maxwell's equations and the Lorentz force law. Which term in Maxwell's equations was due to Maxwell himself? What is the name of this term?

   (b) What is meant by (vacuum) magnetostatics and what do Maxwell's equations reduce to in this case?

   (c) In two dimensions, show that $\phi = \log |\mathbf{r}|$ satisfies

   $$\nabla^2 \phi = K\delta(\mathbf{r}),$$

   where K is a constant to be determined and $\mathbf{r}$ is the two-dimensional position vector.

   (d) A vacuum has two thin infinitely long straight parallel wires a distance a apart, each of which carries a constant current I in the same direction. Starting from Maxwell's equations and the Lorentz force law together with the result of the previous part (1c), find the magnetic field generated by each wire, and the force per unit length between these wires and give the direction of the force.

2. In the electrostatic limit, starting from the vacuum versions of Maxwell's equations and the Lorentz force law, derive

   (a) the energy of a configuration of point charges q_i at positions $\mathbf{r} = \mathbf{a}_i$, for $i = 1, 2, \cdots, N$, by bringing each particle in turn from infinity.

   (b) the energy required to move a charge q starting a distance a from a plane conducting surface to an infinite distance from the conducting surface. (Hint: it might be helpful to find the force on q first.)

3. (a) State the electromagnetic media form of Maxwell's equations in differential form, giving the definitions of the fields $\mathbf{D}$ and $\mathbf{H}$. What are the physical interpretations of the polarization field $\mathbf{P}$ and magnetization field M?

   (b) Determine the fields $\mathbf{H}$ and $\mathbf{B}$ everywhere for a system consisting of a uniformly magnetized ball of radius a with constant magnetization $\mathbf{M}_0$.

4. A conducting disk of radius b is rotated at angular velocity ω about its axis, which we take to be the z axis. The spindle on which it rotates is non-conducting and of radius a and runs through a hole at the centre of the disk of the same radius. There is a strong magnetic flux density B that is constant and uniform in the z direction.

   (a) Sketch the system described above and label your sketch.

   (b) By considering the forces on the electrons in the disk and assuming that the total force on these electrons is zero in equilibrium, find the strength and direction of the electric field E within the disk. Non-electromagnetic forces can be ignored.

   (c) Hence or otherwise, find the electric potential difference (voltage) between the rim of the disk and the edge of the spindle.

   (d) What might be an application of such a device?

5. Consider an electromagnetic plane wave in a simple conductor of constant and uniform conductivity σ. Throughout this question assume that all fields are proportional to $\exp(i(\mathbf{k}\cdot\mathbf{r}-\omega t))$, where the real part represents the physical field, and ω is real.

   (a) Starting from Maxwell's equations and writing $\mathbf{k}$ in the form $\mathbf{k}=(k_r+ik_i)\hat{\mathbf{k}}$, where $\hat{\mathbf{k}}$ is a real unit vector, and k_r and k_i are real, find the exact expressions for $k_r(\omega)$ and $k_i(\omega)$. Explain carefully how you choose the correct root of the quadratic equation that appears in this calculation.

   (b) Find the lengthscale on which this wave decays, and give the standard name for this lengthscale.

6. (a) Starting from the vacuum versions of Maxwell's equations, prove the following *variation* of Poynting's theorem in a vacuum, which states that

$$\frac{\partial U}{\partial t}+\nabla\cdot\mathbf{S}+\mathbf{E}\cdot\mathbf{J}=0,$$

   where

$$U=\frac{1}{2}\varepsilon_0|\mathbf{E}|^2+\frac{1}{2\mu_0}\mathbf{A}\cdot(\operatorname{curl}\mathbf{B}),$$

$$\mathbf{S}=\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}+\frac{1}{2\mu_0}\frac{\partial}{\partial t}(\mathbf{A}\times\mathbf{B}),$$

   and $\mathbf{A}$ is the vector potential of $\mathbf{B}$.

   (b) What is the physical interpretation of Poynting's theorem?