UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Eng.* *M.Sci.*

Mathematics B45: Discrete Mathematics for Computer Scientists

COURSE CODE	:	**MATHB045**
UNIT VALUE	:	**0.50**
DATE	:	**02-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

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TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Show using a truth table that for any sets X, Y and Z,

$$\big((X \cup Y) \cap (Y \cup Z)\big) \cup X = X \cup Y.$$

Hence shade the set $\big((X \cup Y) \cap (Y \cup Z)\big) \cup X$ on a Venn diagram.

(b)

Say what is meant for a set X to be *countable.*

Show that the set $\mathbb{Z}$ of integers is countable.

2. (a) Define the terms *left inverse*, *right inverse* and *2-sided inverse* of a function $f : X \to Y$.

Define the terms *injective*, *surjective* and *bijective.*

(b) Let $f : X \to Y$ be any function. Prove that if h is a left inverse of f and k is a right inverse of f then $h = k$.

(c) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$, which is surjective but not injective.

Write down two distinct right inverses of your function f.

3. (a) Consider the following permutations:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 3 & 5 & 2 & 4 & 6 & 10 & 7 & 8 & 9 \end{pmatrix},$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 1 & 6 & 7 & 5 & 10 & 9 & 8 \end{pmatrix}.$$

Calculate $\sigma\tau$, $\tau\sigma$ and σ^{-1}.

Write σ and τ in disjoint cycle notation.

Find the orders of σ and τ.

Calculate σ^{1000} and τ^{38}, writing your answers in functional notation.

(b) Calculate the signatures $\epsilon(\sigma)$ and $\epsilon(\tau)$.

4. (a) Let G be a set and $*$ a binary operation on G. State the conditions for $(G, *)$ to be a group.

(b) State Lagrange's Theorem. Use this theorem to prove the following

(a) Suppose that G is finite. Then for any element g in G, the order of g divides the order of G.

(b) Suppose that G is finite and its order is a prime number. Then G is cyclic.

(c) Write down an element x of order 6 in S_5 and write down the cyclic subgroup of S_5 generated by x.

5. (a.) (i.) Solve the congruence:

$$3x \equiv 4 \bmod 7.$$

(ii.) Show that $3x \equiv 4 \bmod 12$ has no solutions.

(b) Define the Euler function φ and calculate $\varphi(20)$ and $\varphi(25)$.

(c) Calculate

(i). $11^{801} \bmod 20$ and $3^{-1} \bmod \phi(20)$.

(ii). Solve the equation $x^3 \equiv 3 \bmod 20$.

6. (a) Find all real solutions to the following simultaneous equations:

$$\begin{array}{ccccccccc} w & + & 2x & + & 2y & + & 2z & = & 7, \\ 2w & + & x & + & 2y & + & 2z & = & 7, \\ 2w & + & 2x & + & y & + & 2z & = & 7. \end{array}$$

(b) Give an LU decomposition of the following matrix:

$$\begin{pmatrix} 1 & -2 & 5 \\ 0 & 2 & 3 \\ 2 & -4 & 7 \end{pmatrix}.$$

(c) Calculate (i) the determinant, and (ii) the adjoint and the inverse of the following matrix:

$$\begin{pmatrix} 2 & 4 & 3 \\ 0 & 1 & -1 \\ 3 & 5 & 7 \end{pmatrix}.$$

7. (a) Define the terms *eigenvalue* and *eigenvector* of an $n \times n$ matrix.

(b) Find the eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 4 \end{pmatrix}.$$

(c) Hence find a simple formula for

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}^n.$$

END OF PAPER