UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics B46: Mathematics and Statistics for Computer Scientists

COURSE CODE	:	**MATHB046**
UNIT VALUE	:	**0.50**
DATE	:	**02-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

03-C0903-3-90

TURN OVER

There are two sections. Full marks may be obtained by answering **five** *questions, but no more than three questions from a single section will count.*

Statistical tables are provided

The use of an electronic calculator **is** *permitted in this examination.*

Section A: Use a separate answer book for this section

1. Specifications for a certain type of printer ribbon state a mean breaking strength of 85 pounds. In order to monitor the manufacturing process, 41 pieces of ribbon are sampled at random and are tested. The following data are obtained:

breaking strength (in pounds)	81-82	82-83	83-84	84-85	85-86	86-87	87-88	88-89
frequency	2	7	10	11	7	3	0	1

(a) Calculate the sample mean and the sample variance of the above breaking strength data.

(b) Find the median and the mode of these data.

(c) Calculate the range and the interquartile range of these data.

(d) Perform an appropriate statistical hypothesis test, at a significance level of 1%, to assess whether the true mean breaking strength of the printer ribbon is actually below 85 pounds. State your conclusion clearly.

2. *(Recall that an exponentially distributed random variable X with mean $E(X) = 1/\lambda$ has probability density function $f(x) = \lambda e^{-\lambda x}$ for $x > 0$; its variance is $Var(X) = 1/\lambda^2$.)*

A manufacturer of light bulbs has two production lines, A and B, producing apparently identical light bulbs. However, production line A is more modern and produces 70% of the company's light bulbs. Production line B is slower, producing 30% of the company's light bulbs. Each bulb from production line A has an exponentially distributed lifetime, with a mean of 300 days. Each bulb from production line B also has an exponentially distributed lifetime, but with a mean of 200 days. The lifetimes of different bulbs are independent.

(a) From the probability density function of the exponential distribution, deduce that, for $x > 0$, the probability that a randomly chosen bulb from production line A lasts longer than x days is given by $e^{-x/300}$.

(b) Calculate the overall proportion of light bulbs produced by the manufacturer that last longer than 350 days.

(c) Suppose that a randomly sampled light bulb produced by the manufacturer still works after 350 days. Calculate the (conditional) probability that this bulb came from production line A.

(d) Consider a lot of 100 light bulbs from production line A, and denote the average of their lifetimes by $\bar{X}$. Calculate the mean and the standard deviation of $\bar{X}$. Approximately calculate $P(\bar{X} > 350)$.

3. (a) The London Tea Company sells tea in boxes of 200 tea bags. Assume that each tea bag has a probability of 5% of being underweight, independently of all other tea bags. Denote by X the number of underweight tea bags in a randomly sampled box. Name the distribution of X and find its mean and variance. Approximately calculate the probability that more than 15 tea bags in the box are underweight.

(b) The tea bags are produced using a machine which automatically fills the tea bags with tea. The average amount of tea put into the tea bags is controlled by a dial on the machine: the amount of tea (in grammes) dispensed into any tea bag has a normal distribution with mean μ equal to the amount shown on the dial and standard deviation $\sigma = 0.3$ grammes, independent of all other tea bags. A tea bag is 'underweight' if it contains less than 2 grammes of tea. To what weight μ should the dial be set in order to ensure that only 5% of all tea bags produced are underweight?

4. In a box of 5 unlabelled diskettes there are 2 that contain a file with this exam paper. Diskettes are drawn at random from this box, without replacement, until the box is empty.

(a) Denote by N the number of diskettes drawn until the first diskette is found that contains the exam paper.

(i) State with a reason whether or not N has a geometric distribution.

(ii) Find $P(N = 2)$.

(iii) Calculate the mean and the variance of N.

(b) Calculate the probability that the 2nd **and** the 3rd diskettes drawn both contain the exam paper.

(c) Calculate the probability that the 2nd **or** the 3rd diskette drawn contains the exam paper.

(d) Given that of the first 2 diskettes drawn, at least one contains the exam paper, calculate the (conditional) probability that the 1st diskette drawn contains the exam paper.

Section B: Use a separate answer book for this section

5. (a) Prove, from the definition of the derivative, that the derivative of the product of two functions $(fg)'$ equals $f'g + g'f$.

   (b) Define the function $\arcsin(x)$, stating for which values of x your function is defined. Explain why the derivative of $\arcsin(x)$ is $1/\sqrt{1-x^2}$.

   (c) Find all solutions of the differential equation $y' = y + e^x \tan x$.

6. (a) Find the following limits:

   (i) $\lim_{x\to 0} \frac{\cos(3x)-1}{\sin^2 x}$;

   (ii) $\lim_{x\to 0} \frac{\sin(5x)-5\sin x}{\tan(x)-x}$;

   (iii) $\lim_{x\to +\infty} x(e^{1/x}-1)$.

   (b) Find the Taylor series expansion of $f(x) = \sin x \cos x$ at the point $a = 0$.

7. (a) Find all stationary points of the function $f(x,y) = e^{x^2+y}(x+y)$ and classify them as local maxima, local minima or saddle points.

   (b) Find the volume of the 3-dimensional region which consists of the points satisfying the following inequalities: $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le e^{x+y}(x+y)$.

8. Define $f(x)$ in the following way: $f(x) = |x|$ when $-\pi \le x \le \pi$ and f is periodic with period 2π.

   (i) Draw the graph of the function f.

   (ii) Find the Fourier series expansion of f.

   (iii) Compute $\sum_{l=1}^{\infty} \frac{1}{(2l-1)^2}$.