UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc.   *M.Sci.*

Mathematics M253: Computational Methods

COURSE CODE	:	**MATHM253**
UNIT VALUE	:	**0.50**
DATE	:	**26-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

06-C1053-3-80

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) (i) What is the base 2 (binary) representation for 2.875?
(ii) What is the base 2 representation of 2/3?

(b) Estimate the amount of memory required to store the following arrays, if integers are stored using 8-bit format and reals as 32-bit floating point numbers. Show your working.

```
INTEGER:: VASTSPACE(100,1000,20)
REAL:: HUGEARRAY(-1000:1000,400)
```

(c) Express the integers -15 and 81 in 8-bit binary according to the 2's complement method. Show how the two binary numbers may be added to find the 2's complement representation of 66.

(d) Find five errors in the following FORTRAN program

```
PROGRAM MYMISTAKE
IMPLICIT NONE
INTEGER:: A(3)=0, B(3)=(1,2,3)
LOGICAL:: FACT
DO I=1,3
   A(I)=B(I+1)
   FACT= A(I)*B(I)
   IF (FACT .eqv. 0) EXIT
END DO
PRINT *, FACT IS UNTRUE, FACT
END PROGRAM MYMISTAKE
```

(e) What will be printed when the following DO loop runs?

```
INTEGER:: COUNT
LOGICAL:: TRUTH
DO COUNT=13,1,-3
   TRUTH=( (COUNT/2) > (2+(-1)**(COUNT-1)) )
   PRINT *,COUNT, TRUTH
END DO
```

2. A FORTRAN program uses the following data type to store times in units of hours, minutes and seconds

```
TYPE TIME
  INTEGER:: HOURS, MINUTES, SECONDS
END TYPE TIME
```

(i) Write a FORTRAN function ADDTIMES that has two TYPE(TIME) variables as inputs and returns a TYPE(TIME) variable containing the sum of the two times in hours, minutes and seconds.

(ii) Write a FORTRAN function MULTTIMES that has a single TYPE(TIME) variable and an integer M as inputs and returns a TYPE(TIME) variable that is M times longer than the input period.

(iii) Write down the FORTRAN commands necessary to overload the operators '+' and '*' so that

SUMTIMES = TIME1+TIME2 and FOURTIMES = 4*TIME1

use ADDTIMES and MULTTIMES to perform the relevant operations.

3. (a) A function $y(x)$ is known only on a finite set of points (x_i, y_i), where $i = 0, ..., N$ and the x_i are evenly spaced. Estimate the gradient dy/dx at a point x_i using

(i) a left-sided derivative

(ii) a centred derivative.

State which estimate is more accurate as the grid-spacing $h = x_{i+1} - x_i$ tends to zero.

(b) The Euler method for solving the first order ordinary differential equation

$$\frac{dy}{dx} = F(x, y)$$

is given by

$$x_{n+1} = x_n + h, \qquad y_{n+1} = y_n + hF(x_n, y_n),$$

and can be derived from the left-sided derivative defined above. Write down the mid-point scheme for the same equation, and explain its connection with the centred derivative above.

(c) Write a FORTRAN program to find $y(1)$ using the midpoint method if

$$\frac{dy}{dx} = \cos\left\{x^2 y\right\} \qquad y(0) = 0.$$

Let the step size $h = 0.01$, and write out the output $y(1) = y_{100}$ to the screen giving four decimal places.

4. (a) Show, using a diagram to illustrate your argument, how Newton's method can be used to improve upon an estimate x_n to a root of a nonlinear equation

$$f(x) = 0,$$

where $f(x)$ is a known continuously differentiable function. Write down an equation for the new estimate x_{n+1}. If this equation is to be used to generate a sequence $\{x_n\}$ that may converge to the root, how many initial guesses are necessary?

(b) Give a reason why the above sequence may not converge to a root of $f(x)$.

(c) Find a polynomial that interpolates the set of points $\{(1,15),(4,12),(6,20)\}$.

(d) The random variable X is uniformly distributed on [0,1]. Describe, using FORTRAN commands if desired, how to use X to generate

(i) An integer random variable Z with identical distribution to the number of 'heads' appearing when four fair coins are flipped.

(ii) A real random variable Y with probability density function given by

$$P(Y) = \frac{3}{Y^4}, \qquad 1 < Y < \infty.$$

5. Write FORTRAN functions or subroutines to evaluate the following:

(i) The real-valued scalar function

$$f(x) = x|x|,$$

defined for all real x.

(ii) The real-valued vector field given by

$$\mathbf{F}(\mathbf{x}) = (\mathbf{k} \times \mathbf{x}) \times \mathbf{x},$$

as a function of the real-valued position vector $\mathbf{x} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(iii) The integer function M defined recursively for all positive integers n by

$$M(n) = \begin{cases} n - 10 & \text{if } n > 100 \\ M(M(n+11)) & \text{if } n \leq 100, \end{cases}$$

Evaluate $M(99)$.

6. (a) Describe, in words supplemented by diagrams if desired, an algorithm to arrange an unsorted array of N integers into an ordered binary tree.

(b) Describe an algorithm to sort the resulting ordered binary tree into an ordered array of integers.

(c) What is the order of the algorithm you have described in (i) and (ii) (i.e. the functional dependence of the expected number of operations on N)? Describe your reasoning carefully. How does this compare to a direct sort of the array involving one integer at a time?