UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics M253: Computational Methods

COURSE CODE	:	**MATHM253**
UNIT VALUE	:	**0.50**
DATE	:	**28-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) (i) What is the base 2 (binary) representation for 61.75?
   (ii) What is the base 2 representation for 0.6?

   (b) Suppose a computer system uses the 2's complement method to represent integers, and integers are allocated 8 bits each. What are the lowest and highest numbers allowed (express these in both base 10 and base 2)? What happens if you add one to the highest number?

   (c) Find five errors in the following Fortran Code:

```
PROGRAM UNFORTUNATE
IMPLICIT NONE
INTEGER :: J, K, 4, HEIGHT(3)
J = K = 3
C = J**2
DO J = 1, 3
IF (J /= 2) THEN
        WRITE(2, *) J, HEIGHT(J)
END DO
END IF
END PROGRAM UNFORTUNATE
```

   (d) What will be printed when the following DO loops run?

   (i)
```
DO OUTER = 1, 3
    DO INNER = 1, OUTER
        WRITE(*, *) INNER + OUTER
    END DO
END DO
```

   (ii)
```
COUNT = 1
A = 1
DO
IF (COUNT > 4) EXIT
    A = A * COUNT
    COUNT = COUNT + 1
WRITE(*,*) "A = ", A
END DO
```

2. (a) Write a function MEAN which takes an array of real numbers $A(i)$, $i = 1, \ldots, N$ and calculates their mean (average) value $\overline{A}$. The first few lines should be

FUNCTION MEAN(ARRAY)
REAL, INTENT(IN) :: ARRAY(:)
REAL :: MEAN

(Hint: The size of an array can be found by using the SIZE(ARRAY) function).

(b) Next, write a SUBROUTINE STATS(ARRAY, AVERAGE, STDDEV) which takes an array of real numbers $A(i)$, $i = 1, \ldots, N$ and returns both the average $\overline{A}$ and standard deviation $\sigma(A)$. This subroutine should use the function MEAN from part (i) to calculate $\overline{A}$. The standard deviation is defined as

$$\sigma(A) = \left(\frac{1}{N} \sum_{i=1}^{N} (A(i) - \overline{A})^2 \right)^{1/2}.$$

(c) Write a main program which prompts the user for a set of 10 numbers. The program then reads the 10 numbers and uses the STATS subroutine to calculate the mean and standard deviation. These results are then printed to the screen.

3. (a) Consider a differential equation $\mathrm{d}y/\mathrm{d}x = G(x, y)$. Briefly describe the Euler method for solving this equation.

(b) The recurrence relation for the mid-point method is

$$\begin{aligned} \widetilde{x}_n &= x_n + \frac{h}{2} \\ \widetilde{y}_n &= y_n + \frac{h}{2} G(x_n, y_n) \\ x_{n+1} &= x_n + h \\ y_{n+1} &= y_n + h\, G(\widetilde{x}_n, \widetilde{y}_n). \end{aligned}$$

Briefly describe why the midpoint method may be more accurate than Euler's method.

(c) Write a Fortran program which uses the midpoint method to find $y(1)$, where $y(0) = 0$. Let $h = 0.01$, and

$$G(x, y) = \cos(2\pi(x + y)).$$

The values of $y_1, y_2, \ldots, y_{100}$ should be stored in an array, and printed out ten numbers per line to the file "RungeKutta.dat".

4. (a) Suppose a program stores the positions of a set of points $x_i, i = 0, ...n$ and the values of a function $f(x)$ at those points, $f_i = f(x_i)$. Write down an approximate expression for $df/dx(x_i)$ which uses the data x_i, x_{i+1}, f_i, and f_{i+1}. Call this approximation f'_{+i}. Write down a similar expression for f'_{-i} using x_i, x_{i-1}, f_i, and f_{i-1}. If $f(x) = x^m$, what is the maximum value of m for which these derivatives are exact?

   (b) Find a third expression for $df/dx(x_i)$ which is a better approximation than f'_{+i} and f'_{-i}. For what maximum order of polynomial (value of m) is this expression exact?

   (c) The Subroutine RANDOM_NUMBER(X) gives the real variable X a random value with a uniform distribution between 0 and 1. Using RANDOM_NUMBER(X), or otherwise, how can you give an integer variable K a random integer value between -10 and 10 (i.e. $-10 \leq K \leq 10$)?

   (d) Suppose the real variable X is randomly generated with a uniform distribution between 0 and 1. We wish to create a random variable Y with probability distribution

$$P(Y)\,\mathrm{d}Y = \sin Y\,\mathrm{d}Y, \qquad 0 < Y < \frac{\pi}{2}.$$

   Find the function $Y(X)$ which will give this distribution.

5. (a) Suppose the function $f(x)$ has a root a where $f(a) = 0$. Write a program which uses the bisection method to find this root. The program should prompt the user for two points $XLOW$ and $XHIGH$ on either side of the root, and also for the maximum error $|x - a|$ allowed for an estimate of the root. The program contains a FUNCTION F(X) which calculates the values of $f(x)$. For this question, let

$$f(x) = \cos(2\pi x) - 2x.$$

   Once the program gets within the maximum error, it should print out its approximation for a. It should also print out the number of iterations (bisections) it has done.

   (b) For the above function, there is one real root. Specify values $XLOW$ and $XHIGH$ on either side of this root.

   (c) If we let $x_{-1} = XLOW$ and $x_0 = XHIGH$, then the bisection method generates a sequence of numbers $x_1, x_2, \ldots, x_N$. If $\Delta = XHIGH - XLOW$, determine how many iterations N are needed to find the root with an error

$$|x_N - a| < \frac{\Delta}{1000}.$$

6. Suppose a playing card is represented by the data type

```
TYPE CARD
    INTEGER :: VALUE
    INTEGER :: SUIT
END TYPE CARD
```

Here the suits *spades, hearts, diamonds*, and *clubs* are represented by numbers 1,2,3,4.

(a) Write a subroutine which shuffles a deck of 52 cards, by dividing the deck in 2, then interleaving each card from the first half with a card from the second half. The initial deck is represented by an array UNSHUFFLED of 52 cards. The subroutine returns an array called SHUFFLED. The first few lines may read

```
SUBROUTINE SHUFFLE(UNSHUFFLED, SHUFFLED)
TYPE(CARD), INTENT(IN) :: UNSHUFFLED(52)
TYPE(CARD), INTENT(OUT) :: SHUFFLED(52)
```

(b) Write a subroutine which prints out the contents of a hand of 5 cards. The first few lines may read

```
SUBROUTINE PRINTHAND(HAND)
TYPE(CARD), INTENT(IN) :: HAND(5)
```

A typical output of this subroutine might be:

7 of Spades
13 of Hearts
4 of Spades
2 of Clubs
12 of Diamonds

(c) How might you modify the above subroutine to write out "Jack" instead of 11, "Queen" instead of 12, and "King" instead of 13?