

All questions may be attempted but only marks obtained on the best **four** solutions will count.

The use of an electronic calculator is **not** permitted in this examination.

1. (a) (i) What is the base 2 (binary) representation for 34.625?
(ii) What is the base 2 representation for $1/7$?
- (b) (i) If N equals the number of bits in a megabyte, what is $\log_2(N)$?
(ii) Give a very approximate estimate for the number of bytes needed to store a 300 page book, showing details of your estimate.
(iii) How many bytes are needed to store the following array:
INTEGER :: MATRIX(0:4, 10)

- (c) Find five errors in the following Fortran Code:

```
PROGRAM RIDICULOUS
IMPLICIT NONE
INTEGER :: Y
Y = 4
REAL :: X, Z, TWOZ
Z = X+2
2*Z = TWOZ
DO
COUNT = 1, 4
    Y = Y + COUNT
    IF (Y==3)
        WRITE(*, *) Y
    END IF
END DO
END PROGRAM RIDICULOUS
```

- (d) What will be printed when the following DO loops run?

- (i) A = 2
DO INDEX = 10, 6, -2
 A = A + INDEX
 PRINT*, A
END DO
- (ii) DO OUTER = 1, 3
 DO INNER = OUTER + 1, 4
 PRINT*, INNER*OUTER
 END DO
END DO

2. Write a program which constructs a logical truth table. First declare logical variables a , b , c , P , and Q . Using a DO LOOP, go through all eight possible true and false combinations of a , b , and c . For each combination, calculate the values of P , and Q , where P and Q represent logical statements:

$$P = (a \text{ and } b) \text{ or not } c,$$

$$Q = ((\text{not } a) \text{ or } c) \text{ and } ((\text{not } b) \text{ or } c).$$

(Fortran syntax differs slightly from the above). The program should then print out the truth table. The output should look like

```
a b c  P Q
T T F  T F
T T T  T T
T F F  T F
T F T  F T
F T F  T F
F T T  F T
F F F  T T
F F T  F T
```

The program should also test whether statements P and Q are equivalent. The program should print a message stating that they are (or are not) equivalent.

3. (a) Write a program which prompts the user to enter a positive integer. It then calculates all the factors of the integer and stores the factors in an array. The program counts the number N of factors and prints out the factors to the screen. The factors should be printed out with 5 numbers per line. The program then prints out the number of factors. If the number is prime, the program should print a message telling the user that the number is prime.
- (b) A *perfect number* is a number which equals the sum of its factors. For example, $28 = 1+2+4+7+14$. What Fortran lines would you add to the program which would print out "Perfect Number!" when a perfect number is detected?
- (c) Two integers x and y are *relatively prime* if they share no factors in common (other than 1). Describe in words (or write additional Fortran commands) how you would modify the program in part (a) to take two numbers and decide whether they are relatively prime.

4. (a) Suppose N data records need to be processed. Algorithm A takes a time $t_A(N) = N^2 - N$ seconds to complete the job, while algorithm B takes $t_B(N) = 36N$ seconds. For what values of N will algorithm A be more efficient? In order notation, what are the orders of algorithms A and B?
- (b) Briefly describe how a binary search algorithm works. Given N pieces of data, what is the order of $t(N)$?
- (c) Consider a second order differential equation for the function $y(x)$,

$$y''(x) + q(x)y'(x) + r(x)y(x) + s(x) = 0.$$

Show that this equation can be rewritten as a set of two first order differential equations, involving $y(x)$ and a second function $g(x)$.

These two first order differential equations can be combined into a single matrix equation involving the two-vector

$$F(x) = \begin{pmatrix} y(x) \\ g(x) \end{pmatrix}.$$

What is this equation for $F(x)$?

- (d) Given N data points $(x_i, y_i), i = 1 \dots N$, find a unique $(N - 1)$ th order polynomial which passes through those N points.
- (e) The Subroutine RANDOM_NUMBER(X) gives the real variable X a random value between 0 and 1. Using RANDOM_NUMBER(X), or otherwise, how can you give an integer variable K a random integer value between 1 and 50 (i.e. $1 \leq K \leq 50$)?
- (f) Suppose the real variable X is randomly distributed between 0 and 1. We wish to create a random variable Y with probability distribution

$$P(Y)dY = e^{-Y}dY, \quad 0 < Y < \infty.$$

Find the function $Y(X)$ which will give this distribution.

5. (a) Write a function

```
FUNCTION DAYOFYEAR(DAY, MONTH)
INTEGER, INTENT(IN) :: DAY, MONTH
```

that takes as input the day and the month (as integers) and gives the number of the day (i.e. 1 January \rightarrow 1, 3 February \rightarrow 34, 7 March \rightarrow 66). You may ignore leap years.

- (b) Write a subroutine

```
SUBROUTINE TIME(REALTIME, HOUR, MINUTE)
REAL, INTENT(IN) :: REALTIME
INTEGER, INTENT(OUT) :: HOUR, MINUTE
```

which takes a real number REALTIME, which measures time in hours past midnight, and returns the hour and minute (for example REALTIME = 8.75 would give HOUR = 8 and MINUTE = 45).

- (c) The hour of sunrise for a location at latitude ℓ can be approximately calculated from the following formula. Let d be the number of the day in the year, and let the angle ϕ be defined by

$$\begin{aligned}\phi_0 &= \frac{2\pi}{365}(10 + d), \\ \phi &= \phi_0 + 0.04 \sin \phi_0.\end{aligned}$$

Also let

$$\lambda = -0.438 \tan(\pi\ell/180).$$

Then sunrise occurs at a time

$$t = \frac{12}{\pi} \cos^{-1}(\lambda \cos \phi)$$

where t is a real number with units of hours (past midnight).

Write a program which asks the user for the latitude, and then for the day and month. The program then prints out a message stating when sunrise occurs. The sunrise time should be stated in hours and minutes. The Fortran intrinsic function for inverse cosine ($\cos^{-1}(x)$) is ACOS(x).

6. (a) Suppose the authors and titles of a list of books are stored in a file called *'booklist.txt'*. The file contains 1000 lines. On each line is given the author, title, and publication year of a book. The books are not ordered in any way. Write a program `FINDBOOKS` which opens this file, and reads the information into an array or arrays. For full credit, store the information in an array of the following defined data type `BOOK`:

```
TYPE BOOK
  CHARACTER(LEN=20) :: AUTHOR
  CHARACTER(LEN=20) :: TITLE
  INTEGER :: YEAR
END TYPE BOOK
```

The user of the program is then prompted for the name of an author. The program will then print out the titles and publication years of all books written by that author which are in the booklist. If the author is not found, a message should be printed saying that the author is not on the list.

- (b) Write some lines of Fortran which could be inserted into the above program. These lines will find the book with the earliest publication year, and print out its author, title and year. You may assume that this book is unique, i.e. that there is only one book in the booklist from the earliest publication year.