

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C383: Combinatorial Optimisation

COURSE CODE : MATHC383

UNIT VALUE : 0.50

DATE : 08-MAY-06

TIME : 10.00

TIME ALLOWED : 2 Hours

All questions may be attempted but only marks obtained on the best **four** solutions will count.

The use of an electronic calculator is **not** permitted in this examination.

1. a) Describe the algorithm *Heapsort* (*HS*) for sorting n different numbers into increasing order.
- b) Prove that HS uses $O(n \log n)$ comparisons to sort n numbers.
- c) Use HS to order the following list of numbers:

81, 14, 31, 45, 90, 85.

2. a) Define the Fourier transform of a sequence $(a_0, a_1, \dots, a_{n-1})$ of complex numbers.
- b) Describe the *Fast Fourier Transform* (*FFT*) for finding the Fourier transform of $(a_0, a_1, \dots, a_{n-1})$, where $n = 2^r$ is a power of two.
- c) Show that FFT uses $O(n \log n)$ multiplications of complex numbers.
- d) Use the *Inverse Fast Fourier Transform* (*IFFT*) technique to find the polynomial of degree at most 3 which takes the successive values

$4 - 2i, -8i, 1 + 3i, 5 + 6i$

at the 4th roots of unity $1, i, -1, -i$. Explain what you do.

3. a) Define the *chromatic polynomial* $P(k; G)$ of a finite graph G .
- b) Let e be an edge of G . Define the graphs $G - e$ and G/e . Prove that $P(k; G) = P(k; G - e) - P(k; G/e)$.
- c) Deduce that $P(k; G)$ is indeed a polynomial in k , and that its leading coefficient is k^n , where n is the number of vertices of G .
- d) Show that if G is a tree on n vertices, then $P(k; G) = k(k - 1)^{n-1}$.
(Use the fact that a tree on at least two vertices has a vertex of degree 1.)
- e) Find the chromatic polynomial of a pentagon.

4. a) Describe the *Euclidean Algorithm (EA)* for finding the greatest common divisor $d = \gcd(a, b)$ of two natural numbers a and b .
(Assume, here and below, that $a > b$.)
- b) Show that the number d which EA returns is indeed equal to $\gcd(a, b)$.
- c) Prove that if F_k is the largest Fibonacci number less than or equal to b , then EA takes at most k steps to produce d .
- d) Apply EA to find $\gcd(5214, 1518)$, and express it in the form $5214x + 1518y$, for some integers x and y .
5. a) Describe the concept of a *Turing Machine (TM)*. State carefully how a TM is set up, and how it operates. Explain what is meant by a *step* in a TM calculation.
- b) Set up a TM for the following problem:
Given a finite string of entries 0 and 1, determine whether or not the number of entries 1 is a multiple of 3.
Explain *why* the program works.