

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.Sc. M.Sci.

Mathematics C383: Combinatorial Optimisation

COURSE CODE : **MATHC383**

UNIT VALUE : **0.50**

DATE : **05–MAY–04**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

All questions may be attempted but only marks obtained on the best **four** solutions will count.

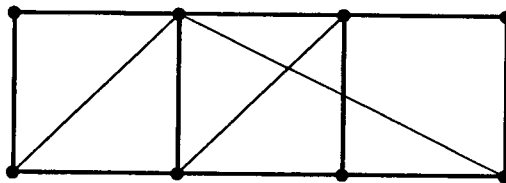
The use of an electronic calculator is **not** permitted in this examination.

1. Describe the algorithm *heapsort* for sorting n numbers into order, and prove that it takes time $O(n \log n)$ to execute.

Use heapsort to order the numbers 73, 13, 29, 41, 88, 78.

2. Describe an algorithm for finding the size of a maximal independent set of vertices of a finite graph G , and state (without proof) its efficiency.

Apply your algorithm to the following graph G :



Any short-cuts used in the calculation should be justified.

3. Describe the MPM algorithm for finding a blocking flow in a layered network Y .
4. Describe the *satisfiability problem* SAT. Assuming that SAT is NP-complete, prove that the graph colouring problem is also NP-complete.
5. Explain how to find the greatest common divisor $d = (a, b)$ of two positive integers a and b , and how to express d in the form

$$d = xa + yb,$$

for some integers x and y . Show that this can be done in time $O(\log_2 m)$, where $m = \max\{a, b\}$.

Find $d := (2002, 561)$, and express it in the form $d = 2002x + 561y$. Count carefully how many steps the calculation takes.