UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C383: Combinatorial Optimisation

COURSE CODE	:	**MATHC383**
UNIT VALUE	:	**0.50**
DATE	:	**06-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

03-C0928-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Let $n = 2^r$ be a power of 2. Define the *Fourier transform* of a sequence $(\xi_0, \dots, \xi_{n-1})$, and show how it can be calculated in time $O(n \log n)$.

   Use the technique of the Inverse Fast Fourier Transform to find the polynomial of degree at most 3 which takes the successive values

   $$4 - 2i,\ -8i,\ -4 - 2i, -8i,$$

   at the 4th roots of unity.

2. Let $\beta_1, \dots, \beta_p \geqslant 0$ with $\sum_{r=1}^{p} \beta_r > 1$, and let the sequence $(x_n \mid n \geqslant 0)$ satisfy a recurrent inequality of the form

   $$x_n \leqslant \sum_{r=1}^{p} \beta_r x_{n-r} + g(n)$$

   for $n \geqslant p$, where $g(n)$ is a polynomial in n. Prove that there is a number $\gamma > 1$ (which should be carefully described) such that $x_n = O(\gamma^n)$.

   The condition that $g(n)$ be a polynomial in n is too strong. Suggest (with proof) a weaker condition which will suffice to give the same estimate for the order of x_n.

3. Define the *chromatic polynomial* $P(k; G)$ of a graph G, and prove that it has the following properties:

   (a) it is a polynomial in k;

   (b) it has leading term k^n, where $n := |V(G)|$;

   (c) its coefficients alternate in sign;

   (d) it has no constant term;

   (e) the coefficient of k^{n-1} is $-|E(G)|$.

   Describe an algorithm for calculating $P(k; G)$, and estimate its efficiency.

4. Let X be a network, and f a flow in X. Describe how to construct the *layered network* $Y := Y(X, f)$.

   Define the *height* of a layered network Y, and show that, if $Y(p)$ and $Y(p+1)$ are two successive layered networks of X, then $Y(p+1)$ has larger height than $Y(p)$.

5. Describe the euclidean algorithm, for finding the greatest common divisor d of two natural numbers a and b, and expressing d in the form

$$d = xa + yb,$$

   for some integers x and y. Show that this can be done in time $O(\log_2 m)$, where $m = \max\{a, b\}$.

   Illustrate the method with $a = 3498$ and $b = 2442$.

END OF PAPER