UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.A. *B.Sc.* *B.Sc. (Econ)* *M.Sci.*

Mathematics B51A: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51A**
UNIT VALUE	:	**0.50**
DATE	:	**07-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**2 hours**

02-C0903-3-170

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

A table of integrals is attached.

1. (a) If f is a real valued function defined on an open interval containing a, define what it means for f to be differentiable at a.

   (b) Differentiate the following functions:

   (i) $f(x) = (1 - \sin x)(2x^5 + e^{\cos 2x})$;

   (ii) $f(x) = \dfrac{\cos(x^2+1)}{\ln(x^2 - 2x + 4)}$.

   (c) Assuming that the equation $x^5 + 4x^2y + y^2 = 32$ determines a function f such that $y = f(x)$, find y' and find the tangent line to the graph of $x^5 + 4x^2y + y^2 - 32 = 0$ at the point $P(2, 0)$.

2. (a) Let f be a real valued function on a closed interval $[a, b]$. State the Intermediate Value Theorem and the Mean Value Theorem for f.

   (b) Verify the Intermediate Value Theorem for $f(x) = x^2 + 2x + 1$ on $[1, 2]$.

   (c) For $f(x) = x^2 + 2x - 5$, find all numbers c in $[0, 1]$ that satisfy the Mean Value Theorem.

3. Evaluate the following integrals:

   (a) $\int x \cos 7x dx$

   (b) $\int \frac{x^3}{\sqrt{16x^2+64}} dx$

   (c) $\int \frac{7x-15}{x(x-3)} dx$

   (d) Evaluate the integral $\int \frac{1}{x^2\sqrt{1-4x^2}} dx$ by means of hyperbolic substitution.

4. State both the Comparison Test and the Limit Comparison Test for the convergence of infinite series. Use these comparison tests to determine whether the following series converge or diverge.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n5^n}$
   (b) $\sum_{n=1}^{\infty} \frac{7n^2-9}{2e^n(3n+1)^2}$
   (c) $\sum_{n=1}^{\infty} \frac{3}{1+2\sqrt{n}}$
   (d) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2n^2+3}$
   (e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+2}}$

5. (a) Find the real and imaginary parts of the complex numbers:
   (i) $\dfrac{4-5i}{2+3i}$;
   (ii) $\dfrac{(1+i)^{12}}{2-3i}$.

   (b) Find all complex numbers z such that $z^5+1=0$ and indicate your answers on the Argand diagram.
   (c) Solve the equation $z^4+1-i\sqrt{3}=0$.
   (d) For $z=\cos\theta+i\sin\theta$, expand $(z+z^{-1})^4$ and obtain an expression for $\cos^4\theta$ in terms of $\cos 2\theta$ and $\cos 4\theta$.

6. a) Show that $f_{xy}=f_{yx}$ for $f(x,y)=2x^2+3x-5xy+2y-y^2$.

   b) Use differentials to approximate the change in
   $$w=2x^2-4x^3y^2+2y^2$$
   if (x,y) changes from $(1,1)$ to $(1.01,1.03)$.

   c) Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ for $w=2u\cos 3v$, $u=4x^2+5y^3$ and $v=3x^3y^2$.

   d) If $z=f(x,y)$ satisfies the equation
   $$2x^2z^2+5xy^2-2z^2+6yz+7=0,$$
   find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

7. A rectangular box with no top and having a volume of 12 cubic metres is to be constructed. The cost per square metre of the material to be used is 4 pounds for the bottom, 3 pounds for two of the opposite sides, and 2 pounds for the remaining pair of opposite sides. Find the dimensions of the box that will minimize the cost.

END OF PAPER

Table of Integrals

Basic Forms

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int \csc u \cot u\,du = -\csc u + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u + a}{u - a}\right| + C$$

$$\int u^n\,du = \frac{1}{n + 1}u^{n+1} + C, \quad n \neq -1$$

$$\int e^u\,du = e^u + C$$

$$\int \sin u\,du = -\cos u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \sec u \tan u\,du = \sec u + C$$

$$\int \tan u\,du = \ln|\sec u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u - a}{u + a}\right| + C$$

Hyperbolic Forms

$$\int \sinh u\,du = \cosh u + C$$

$$\int \tanh u\,du = \ln\cosh u + C$$

$$\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$$

$$\int \operatorname{sech}^2 u\,du = \tanh u + C$$

$$\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$$

$$\int \cosh u\,du = \sinh u + C$$

$$\int \coth u\,du = \ln|\sinh u| + C$$

$$\int \operatorname{csch} u\,du = \ln|\tanh \tfrac{1}{2}u| + C$$

$$\int \operatorname{csch}^2 u\,du = -\coth u + C$$

$$\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$$