UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C394: Biomechanics

COURSE CODE	:	**MATHC394**
UNIT VALUE	:	**0.50**
DATE	:	**18-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best ***four*** *solutions will count.*
The use of an electronic calculator is ***not*** *permitted in this examination.*

1. In each of the following, state clearly any additional assumptions that you make, and, where possible, indicate how good you think that your assumptions are. Where necessary assume that the metabolic rate P of a mammal of mass M is given approximately by $P \propto M^{\frac{3}{4}}$.

   (a) Suppose that two mammals, each of mass M, fight a similar-shaped mammal of mass αM, and that the outcome of the fight is that one side is killed and the other is unharmed. If the mammal of mass αM wins, he can live off the meat of the other mammals for a time T. Determine how long, in terms of α and T, the two mammals of mass M could live off the meat of the mammal of mass αM, if they were to win.

   (b) When a gibbon swings through trees, it has two modes of movement depending on its speed: (i) continuous contact, i.e. at least one hand is holding a branch, and (ii) alternate swing and flight phases. Determine the scaling law for the transition speed U between these two modes of motion in terms of the size of the animal. Why is the continuous-contact motion of a gibbon through trees more efficient than a similar-sized biped walking?

   (c) Determine the scaling law for the breathing rate of a mammal of mass M.

2. (a) Describe the tracheal system of an insect in a few sentences.

   (b) Write down, without proof, the oxygen flux equations for a pipe in the body network.

   (c) Consider part of an insect's tracheal system that is a tetrahedron where the pipes are its edges, and all the pipes are exactly the same, in a system with no advection. Suppose exactly one vertex O is connected to the atmosphere where the oxygen concentration is ϕ_∞, and determine the oxygen concentration at the other vertexes. By inspection, identify the point with the lowest oxygen concentration, and determine the oxygen concentration at that point.

3. (a) State, in a few sentences, some of the main adaptations for flight in most birds that are capable of flight.

   (b) The equation for the drag is usually written in the following form:

$$D = D_f + D_i, \tag{1}$$

   where

$$D_f = \tfrac{1}{2}\rho U^2 S C_{Df}, \quad D_i = KL^2/(\tfrac{1}{2}\rho U^2 b^2). \tag{2}$$

   Define all the symbols used in the above equations. Sketch the function D against U.

   (c) Develop the theory of the bounding flight of some small birds, determining (i) when bounding flight is possible, and (ii) under what conditions bounding flight is more efficient than continuously flapping flight. Find an expression for the optimal flapping fraction f for bounding flight.

   (d) Determine mathematically, using scaling arguments, how many flaps of the wings a (flight-capable) bird needs to take off on level ground in still air as a function of mass M.

4. (a) What is meant by the term 'creeping gait' when applied to a $2n$-legged animal?

   (b) Describe a full cycle of a creeping gait for a 6-legged animal (say an insect) walking in a straight line; use diagrams wherever possible to aid your explanation.

   (c) Define the standard numbering system for the legs of a 4-legged animal. Show that there is a creeping gait with stepping sequence 1423 for a 4-legged animal walking in a straight line; use diagrams wherever possible to aid your explanation.

   (d) Determine a creeping gait for a 4-legged animal to turn on the spot, using diagrams wherever possible to describe the stepping sequence.

   Hint: For simplicity, you may assume that the feet of the animal are placed on a circle with the centre of mass of the animal always directly over the centre of the circle.

5. (a) Define the term *chemotaxis*, and in a few sentences describe the motion of a bacterium on a microscopic scale.

   (b) Explain in a few sentences, in what limit it is reasonable to approximate the motion of a bacterium by a stochastic differential equation.

   (c) Consider the motion of a bacterium in a two-dimensional shear flow governed by the Itô stochastic differential equations

$$dX_t = udt + \sigma dB_t, \qquad dY_t = \gamma X_t dt + \sigma dW_t,$$

   where B and W are independent standard one-dimensional Brownian motions, and u, σ and γ are constants. Find $\mathrm{E}[Y_t]$ and $\mathrm{E}[Y_t^2]$ for a bacterium starting at time $t = 0$ from the origin.