# UNIVERSITY COLLEGE LONDON

University of London

## EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

*B.Sc.* *M.Sci.*

**Mathematics C394: Biomechanics**

COURSE CODE : **MATHC394**

UNIT VALUE : **0.50**

DATE : **09-MAY-05**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

05-C0942-3-40



**TURN OVER**

*All questions may be attempted but only marks obtained on the best* ***four*** *solutions will count.*
*The use of an electronic calculator* ***is*** *permitted in this examination.*

1. In each of the following state clearly any additional assumptions that you make, and where possible indicate how good you think that your assumptions are. Where necessary assume that the metabolic rate $P$ of a mammal of mass $M$ is given approximately by $P \propto M^{\frac{3}{4}}$.

   (a) Consider two people of exactly the same shape and size, and suppose that their top speeds on solo bicycles on level ground are $U_1$ and $U_2$. What is the highest speed that we could expect if these two people cycled together on a tandem (i.e. a two-person bicycle)? How would the top cruising speed on level ground of solo cyclists be expected to depend on the riders' heights? How would the terminal solo free-wheeling speed down a steep hill be expected to depend on the riders' heights (where the gradient of the hill is assumed to be the same for each rider)?

   (b) Find the scaling law for the gestation (i.e. pregnancy) period of similar shaped mammals.

2. A yucca plant can be modelled as a long tapering trunk with a large number of long leaves all of the same length attached to the trunk at constant number density. Assuming that the cross-sectional area of the trunk is proportional to the static load, find the equation for the radius of the trunk, assumed circular, as a function of the distance from the top of the plant, stating any additional assumptions clearly.

3. (a) State in a few sentences some of the main adaptations for flight in most birds that are capable of flight.

   (b) The equation for the drag is usually written in the following form:

$$D = D_f + D_i, \tag{1}$$

   where

$$D_f = \tfrac{1}{2}\rho U^2 S C_{Df}, \quad D_i = KL^2/(\tfrac{1}{2}\rho U^2 b^2). \tag{2}$$

   Define all the symbols used in the last equation. Sketch the function $D$ against $U$.

   (c) Show mathematically that a glide at angle $\theta$ is unstable for $U$ below a critical speed; you may assume, for simplicity, that the motion remains in a straight line. Determine this critical speed.

   (d) Show that a bird soaring at constant height in a thermal with updraught speed $v$ needs to keep its speed $U$ in the range

$$\left(\frac{mgv}{\rho S C_{Df}}\right)^{\frac{1}{3}} < U \leq \left(\frac{2mgv}{\rho S C_{Df}}\right)^{\frac{1}{3}}. \tag{3}$$

4. (a) Describe the tracheal system of an insect in a few sentences.

   (b) Write down, without proof, the oxygen flux equations for a pipe in the body network.

   (c) Suppose that part of the tracheal system of an insect can be modelled by a thin pipe of length $8d_1$ bent round to form a closed loop with four linear pipes of length $d_2$ attached to the looped pipe at equally spaced nodes. Suppose also that all the pipes have the same constant cross-sectional areas. The rate of oxygen absorption is a constant $\mu$ throughout the network. Sketch the network and label it clearly. Assuming that the motion of oxygen is purely diffusive (i.e. no advection) and the oxygen concentration at the open ends of the linear pipes is known to be a constant $\Phi_\infty$, find the rate of oxygen consumption of the whole network and find the oxygen concentrations at the internal nodes.

   (d) By inspection, state where you would expect the lowest oxygen concentration to be, and find the value of the oxygen concentration at this point.

5. (a) Suppose that the left ventricle of a mammal can be approximated as a bag with interior $V$ connected to a straight circular pipe (the aorta) of constant cross-sectional area $A_a$. Suppose that the fluid velocity $U(t)$ in the aorta is a function of time $t$ only. The cross-sectional area of the ventricle in planes with normals parallel to the axis of the aorta has the form

$$A(x,t) = f(x) + g(x)h(t),$$

where $x \in [0, L]$, with $x = L$ at the entrance of the aorta. Find, in their simplest forms, the equations for the rate of change of $x$-momentum of the blood in the ventricle and the momentum flux into the aorta, stating clearly any additional assumptions.

(b) Simplify your results for the previous part of this question when the ventricle is a circular cylinder of fixed length with axis parallel to the axis of the aorta.