UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. M.Sci.

Mathematics C394: Biomechanics

COURSE CODE	:	**MATHC394**
UNIT VALUE	:	**0.50**
DATE	:	**30-MAY-03**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

03-C0931-3-40

TURN OVER

*All questions may be attempted but only marks obtained on the best **four** solutions will count.*
*The use of an electronic calculator **is** permitted in this examination.*

1. In each of the following, state clearly any additional assumptions that you make, and where possible indicate how good you think that your assumptions are. Also make sure you define all your symbols and notation. Where necessary assume that the metabolic rate P of an animal of mass M is given approximately by $P \propto M^{\frac{3}{4}}$.

   (a) A collection of similar shaped grazing animals occupy a homogeneous grassland. Determine how the area grazed per day and the distance walked per day while grazing depends on the mass of the animal.

   If two similar grasslands A and B have the same area, but the grazing animals on A are on average α times the mass of those on B, what is the ratio of the maximum number of animals that A and B can support.

   (b) If a set of similar shaped animals eat periodically, and each time they eat till they feel full, how would the time T between meals depend on size?

   (c) Determine the critical speed for the walking-running transition for a biped on level ground, stating clearly any modelling assumptions. What is the ratio of this critical speed for a particular biped on the Earth to that on the Moon (which has approximately $\frac{1}{6}$ of the gravitational acceleration of Earth)? Comment on the likely consequence for a man moving on the surface of the moon.

2. (a) State in a few sentences some of the main anatomical adaptations that enable birds of flight to fly.

   (b) The equation for the drag on a bird is usually written in the following form:

$$D = D_f + D_i, \tag{1}$$

   where

$$D_f = \tfrac{1}{2}\rho U^2 S C_{Df}, \quad D_i = KL^2/(\tfrac{1}{2}\rho U^2 b^2). \tag{2}$$

   Define all the symbols used in the above equations. Sketch the function D against U. Also, by means of a sketch, show all the forces acting on a gliding bird.

   (c) State, without proof, the condition on U for a stable glide. What is the glide speed U_{md} at which the drag is at a minimum? What is the glide angle θ_{md} corresponding to glide speed U_{md}? What shape would you expect the wings of birds to be that have very small values of θ_{md}? Name an example of such a bird.

3. (a) State clearly the main assumptions for the large-scale motion of bacteria to be well approximated by a diffusion equation.

   (b) Suppose that the probability density function $p(\mathbf{r}, t)$ of one bacterium is well approximated by

$$\frac{\partial p}{\partial t} = D\nabla^2 p,$$

   where $\mathbf{r}$ is the position vector and t is time. If, on a microscopic scale, the motion of the bacterium is observed to be repeated straight-line swimming for a (non-random) time T at constant speed v followed by an instantaneous change of direction sampled from a uniform distribution, find the effective diffusivity D.

   (c) Define the term chemotaxis. Suppose now that the time T is direction dependent, and has the form

$$T = \alpha + \beta \mathbf{k} \cdot \mathbf{n},$$

   where $\mathbf{k}$ is a constant unit vector and $\mathbf{n}$ is a unit vector in the current swimming direction. What is the mean drift velocity of the bacterium?

   State, without proof, how the diffusion equation of part 3b can be modified to include this effect.

4. (a) Describe the tracheal system of an insect in a few sentences.

   (b) Suppose that part of the tracheal system of a caterpillar can be approximated by an infinite internal tube with equally spaced nodes $2\ell_1$ apart along its entire length; at each node there are two side branches of length ℓ_2 with the free ends connected to the outside, where the oxygen concentration is Φ_∞. Suppose that the oxygen absorption rates and the pipe cross-sections are uniform and constant for all the pipes in the system and have the same values for all pipes; also suppose that the oxygen transport is purely diffusive, i.e. there is no advection. Sketch the network, adding any labels that you use in your calculation. Find the oxygen concentration at the internal nodes in terms of Φ_∞, defining carefully any other quantities that you need.

   By inspection, find the point in the network that has the lowest oxygen concentration, and determine the oxygen concentration at this point. Mark this point on your diagram.

   (c) In a sentence describe the main difference between oxygen transport in insects and in spiders.

5. (a) Describe in a few sentences the circulatory system of mammals.

   (b) Derive the linearized pulse-wave equation for an artery, modelled as an elastic tube with cross-sectional area $A(x,t)$, assuming that:

   - The tube properties are independent of x.
   - $|A(x,t) - A_0| \ll A_0$, where A_0 is the mean value of $A(x,t)$.
   - The excess pressure $p \approx \sigma.(A - A_0)$, where the constant σ describes the elasticity of the tube.
   - Blood has constant density ρ, and is conserved.

   State any additional assumptions clearly.

   (c) Hence or otherwise, find linearized wave equations for the pressure and the cross-sectional averaged velocity.

   (d) Derive, in integral form, the momentum equation for a material volume of an incompressible fluid in terms of integrals over its surface (i.e. convert volume integrals to surface integrals). State any additional assumptions that you make clearly. Describe in a few sentences a biological application of this equation, and why in this application the surface-integral formulation might be more useful.