UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M13A: Applied Mathematics 1

COURSE CODE : **MATHM13A**

UNIT VALUE : **0.50**

DATE : **10-MAY-04**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

04-C0951-3-100

TURN OVER

All *questions may be attempted but only marks obtained on the best* **four** *solutions will count. The use of an electronic calculator is* **not** *permitted in this examination.*

1. (i) Four digits are selected simultaneously at random (without repetition) from $\{0, 1, 2, ..., 9\}$. What is the probability that

   (a) the four digits form a run? (e.g. 2,3,4,5)

   (b) they are all greater than 5?

   (c) they include the digit 0?

   (d) at least one is greater than 7?

   (ii) In a workshop, the operation of a machine requires it to be fitted with a component of type A on 18 days of a month which is replaced by a component of type B for the other 12 days. Component A has a .05 chance per day of failing and component B has a .1 chance. The machine fails, and without knowing which type of component was in place on that particular day, an employee writes an order for a replacement component of type B. What is the probability that he made a correct decision?

2. If the probability of success in a single trial is p, show that the probability of exactly m successes in N trials is

$$^{N}C_{m}p^{m}q^{N-m}$$

where $q = 1 - p$.

In a square city, the streets run due North and East, and the intersections are labelled (m, n) where m is the number of blocks East and n the number of blocks North, measured from the South-West corner of the city. Starting from $(0, 0)$, a man walks either in a northerly or easterly direction, his choice at each intersection being decided by rolling a die. If he scores 1 or 2 he moves eastwards; otherwise he moves northwards. Find the probability that, after N moves, he is at the point $(m, N - m)$. Write down the probabilities when $N = 6$ and show that the most likely location is (2,4) with a probability of nearly 1/3.

3. Define the moment about the origin of the force $\mathbf{F}$ acting through the point with position vector $\mathbf{b}$.

Two equal rods of weight w are freely jointed and their free ends are attached by strings to a fixed point. A circular disc of weight W and radius r rests in the angle between the rods, and the whole hangs in a vertical plane. The contact between the disc and the rods is smooth. The length of each rod and of each string is $2a$, and 2θ is the angle between the rods. Show that the tension T in each string is given by

$$2T\cos\theta = 2w + W$$

and that the reaction R between the disc and the rods is $\frac{1}{2}W\text{cosec}\,\theta$ at each point of contact. Show also that r is related to a by the formula

$$r = \frac{2a(3w+2W)}{W}\sin^2\theta\tan\theta.$$

4. A length of rope is in contact with a rough plane curve. The coefficient of friction between them is μ and the weight of the rope may be neglected. Consider the limiting equilibrium of an element AB of rope that is about to slip in the direction from A to B. Suppose that at A, B the tangents to the curve make intrinsic angles $\psi \mp \delta\psi/2$ respectively with a fixed direction, and that the tensions there are $T \mp \delta T/2$. By resolving along and perpendicular to the tangent to the element at the point where the intrinsic angle is ψ, show that $dT/d\psi = \mu T$.

A light rope is in contact with that part of the rough plane curve $y = a\cosh(x/a)$ from the point C where $\sinh(x/a) = -(\ell/a)$ to the point D where $\sinh(x/a) = a/\ell$. Here a and ℓ are positive constants and the coefficient of friction is μ. The rope is about to slip in the direction from C to D. If the tensions at C and D are T_C and T_D respectively, find the ratio T_D/T_C.

5. A ball is thrown vertically upwards with speed V_0. Air resistance provides a drag on the ball of cv^2 per unit mass where v is the speed of the ball and c is a positive constant. Find a differential equation relating v and y, the height of the ball, and show that

$$y = \frac{1}{2c}\log_e[(g + cV_0^2)/(g + cv^2)].$$

What is the maximum height reached by the ball?

The ball now returns to its point of projection. With what speed does it hit the ground?

6. A particle of unit mass moves on a straight line under the action of a force with potential

$$V(x) = \frac{x^2}{x^4 + a^4}$$

where a is a positive constant. Sketch $V(x)$ and write down the energy equation. The particle is projected from the origin in the positive x-direction with speed λ/a where λ is a positive constant. Describe what happens to the particle if $\lambda \geqslant 1$.

If, however, $0 < \lambda < 1$, show that the particle oscillates between the points $x = \pm x_0$ where

$$x_0^2 = \frac{a^2}{\lambda^2}\left\{1 - (1 - \lambda^4)^{1/2}\right\}.$$

If λ is very small, so that the oscillations are small, show that their amplitude is $a\lambda/2^{1/2}$ approximately.

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