UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics M11B: Analysis 2

COURSE CODE : **MATHM11B**

UNIT VALUE : **0.50**

DATE : **17-MAY-05**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

05-C0953-3-190

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Suppose f is defined on an open interval containing a. Give a definition for f to be continuous at a. Give a definition for f to be differentiable at a. Show that, if f is differentiable at a, f must be continuous at a.
   (b) If f is defined on $\mathbb{R}$ and $|f(x)-f(y)| \leqslant |x-y|^2$ for all real x and y, show that f must be constant.
   (c) Suppose f is defined on $\mathbb{R}$, $f(x) = 1$ when $x \neq 0$ and $f(0) = 0$. Is there a function $F(x)$ with the property that $F'(x) = f(x)$ for all real x? (Justify your answer.)

2. What is meant by the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$? Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \ell$, where $0 < \ell < \infty$. Show that $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $\frac{1}{\ell}$. Find the radius of convergence of the following series:
   (a) $x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots$
   (b) $x + 4x^2 + 27x^3 + 16x^4 + \cdots + (2n)^2 x^{2n} + (2n+1)^3 x^{2n+1} + \cdots$
   (c) $1 + 3x + 5^2x^2 + 3^3x^3 + \cdots + 5^{2n}x^{2n} + 3^{2n+1}x^{2n+1} + \cdots$

3. (a) State and prove Cauchy's Mean Value Theorem (you may assume Rolle's Theorem).
   (b) State L'Hôpital's Rule. Use L'Hôpital's Rule to evaluate the following limits:
      (i) $\dfrac{\cos \frac{\pi}{2} x}{x^2 - 1}$ as $x \to 1$,
      (ii) $\dfrac{\log(4-x)}{(x^2-9)^{1/2}}$ as $x \to 3$ from above,
      (iii) $\dfrac{(x-\frac{\pi}{2})^4}{\cos^2 x - \cot^2 x}$ as $x \to \frac{\pi}{2}$.

4. (a) State and prove Taylor's Theorem, giving the Cauchy and the Lagrange form of the remainder.

   (b) Find the series expansion of the following:

      (i) $\log(1+x)$,

      (ii) $\log(1+x^2)$,

      (iii) $\log(1+x+x^2+x^3)$.

5. (a) What is meant by a partition P of a closed interval $[a,b]$? Suppose f is a bounded function on $[a,b]$. Define the Upper Riemann Sum, $U(P,f)$, and the Lower Riemann Sum, $L(P,f)$. What does it mean to say that f is Riemann Integrable?

   (b) Show that if f is monotonic it is Riemann Integrable (You may use a general Theorem. If you do, you must quote it clearly.)

6. State and prove the Fundamental Theorem of Calculus. (You may use a general Theorem. If you do, you must quote it clearly.)

   Find $G'(x)$ in each of the following cases:

   (a) $G(x) = \displaystyle\int_1^x \frac{1}{\exp(t)+1}\,dt$,

   (b) $G(x) = \displaystyle\int_x^{x^2} \frac{1}{\exp(t)+1}\,dt$,

   (c) $G(x) = \displaystyle\int_1^{\int_1^x \frac{1}{\exp(t)+1}\,dt} \frac{1}{\exp(t)+1}\,dt$.

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