UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M11B: Analysis 2

COURSE CODE	:	**MATHM11B**
UNIT VALUE	:	**0.50**
DATE	:	**17-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

04-C0947-3-200

TURN OVER

All *questions may be attempted but only marks obtained on the best* **four** *solutions will count. The use of an electronic calculator is* **not** *permitted in this examination.*

1. (i) Define the derivative $f'(a)$ of a function $f(x)$ at the point a. If f and g are differentiable at a, with $f'(a) \neq 0$, show that $f+g$ and $\dfrac{1}{f}$ are also differentiable at a with

$$(f+g)'(a) = f'(a) + g'(a)$$
$$\left(\tfrac{1}{f}\right)'(a) = -\tfrac{f'(a)}{(f(a))^2}$$

(ii) If $f(x) = x^{\frac{p}{q}}$, $x > 0$, p, q positive integers, show that $f'(x) = \frac{p}{q}x^{\frac{p}{q}-1}$.

2. (i) State and prove Rolle's theorem and state and deduce the Mean Value Theorem.

(ii) Evaluate

a)

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}.$$

b)

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin^3 x - \operatorname{cosec}^3 x}{\left(x - \frac{\pi}{2}\right)^2}.$$

3. (i) Let f be a bounded function on $[a, b]$. Define the upper Riemann integral $\overline{\int_a^b} f(x)dx$ and the lower Riemann integral $\underline{\int_a^b} f(x)dx$ and show that

$$\underline{\int_a^b} f(x)dx \leqslant \overline{\int_a^b} f(x)dx.$$

Give an example to show that there is not always equality.

(ii) Show that if f is a strictly decreasing function on $[a, b]$ then f is Riemann integrable on $[a, b]$.

4. (i) State and prove Taylor's theorem with the Cauchy form of the remainder.

(ii) If f and g are two polynomials such that $f^{(j)}(0) = g^{(j)}(0)$ (j^{th} derivatives at 0) for $j = 0, 1, 2, \ldots$, prove that $f(x) = g(x)$ for all x.

(iii) Give an example to show that (ii) is not always true when f and g are two infinitely differentiable functions.

5. (i) State and prove the fundamental theorem of calculus.

(ii) Evaluate

$$\int_0^1 \frac{dx}{(1+x^2)^{3/2}}.$$

(iii) Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\cos^3 x \, dx}{\sqrt{1+\sin x}}.$$

6. (i) Let f, g be differentiable functions on $[a, b]$. Show that F, defined by $F(x) = f(x)g(x)$, is also differentiable on $[a, b]$. Show further that

$$\int_a^b f(x)g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx.$$

(ii) Evaluate

$$\int_0^{\frac{\pi}{2}} \sin^3 \theta d\theta.$$

(iii) Evaluate

$$\int_1^2 (\log x)^2 dx.$$

END OF PAPER