UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M11A: Analysis 1

COURSE CODE	:	**MATHM11A**
UNIT VALUE	:	**0.50**
DATE	:	**19-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

03-C0939-3-200

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Define what it means for a sequence $(x_n)_{n=1}^{\infty}$ to converge to a limit a.

   (b) Prove that if $x_n \to a$ and $y_n \to b$ as $n \to \infty$ then $x_n + y_n \to a + b$ as $n \to \infty$.

   (c) Prove that if $x_n \to a$ and $y_n \to b$ as $n \to \infty$ then $x_n y_n \to ab$ as $n \to \infty$.

   (d) Determine
$$\lim_{n\to\infty} \frac{(n+1)^{2003} + (n+2)^{2003}}{n^{2003}}.$$

2. (a) Define what it means for a sequence $(x_n)_{n=1}^{\infty}$ to be a Cauchy sequence.

   (b) *State* the Bolzano-Weierstrass Theorem.

   (c) *State and prove* the General Principle of Convergence.

3. (a) Define what it means for a series $\sum_{n=1}^{\infty} x_n$ to be convergent.

   (b) *State and prove* the Comparison Test for series.

   (c) Prove that if $\sum_{n=1}^{\infty} x_n$ is a divergent series of nonnegative real numbers and $y_n \geqslant x_n$ for every n then $\sum_{n=1}^{\infty} y_n$ diverges.

   (d) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

   (e) Investigate the convergence of the series
$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}}.$$

4. (a) Define what it means for a series $\sum_{n=1}^{\infty} x_n$ to converge absolutely.

   (b) Define $\exp(x)$ for a real number x by giving a series.

   (c) *State* the Ratio Test for series.

   (d) Prove that the series for $\exp(x)$ converges absolutely.

   (e) Prove that if x and y are real numbers then $\exp(x + y) = \exp(x)\exp(y)$.

   [You may use a general theorem about multiplication of series, provided that you state it carefully.]

5. (a) Define what it means to say that a function f is continuous at c.

   (b) *State and prove* the Intermediate Value Theorem.

   (c) Suppose $f : [0,1] \to [0,1]$ is continuous. Must there be some $x \in [0,1]$ with $f(x) = x$?

6. (a) Define what it means for a function f to be convex on $[a,b]$.

   (b) *State and prove* Jensen's Inequality for convex functions.

   (c) *State and prove* the AM/GM inequality. [You may assume that $\log(x)$ is concave.]

   (d) Prove that, for $n \geqslant 1$,

$$n! \leqslant \left(\frac{n+1}{2}\right)^n.$$