

**UNIVERSITY COLLEGE LONDON**

University of London

**EXAMINATION FOR INTERNAL STUDENTS**

For The Following Qualifications:-

*B.Eng.*    *M.Eng.*

**Chemical Eng E856: Transport Processes III**

**COURSE CODE            :    CENGE856**

**UNIT VALUE             :    0.50**

**DATE                     :    17-MAY-05**

**TIME                     :    10.00**

**TIME ALLOWED         :    3 Hours**

Answer **FOUR** questions. Each question carries a total of 20 marks each, distributed as shown [ ]

Only the first four answers will be marked.

1.

A long horizontal pipe of circular cross-section and radius  $R$  is filled with a Newtonian liquid of viscosity  $\mu$  and density  $\rho$ . A continuous wire, radius  $xR$  ( $x < 1$ ) is drawn along the pipe axis at a steady velocity  $V$ .

Using the continuity and Navier-Stokes equations in the appended equations of change, derive an equation describing the velocity profile in the liquid. Neglect any end effects. [10]

If the pipe is 20 m long,  $x = 1 \times 10^{-1}$ ,  $V = 3 \text{ m s}^{-1}$  and  $\mu = 2 \times 10^{-1} \text{ Pa s}$  calculate the force required to draw the wire through the liquid. [10]

2.

Outline, with the aid of a sketch, the phenomenon of the boundary layer with reference to the two dimensional flow of a Newtonian fluid along a horizontal, flat plate. [5]

The differential equations describing the flow within the boundary layer along a flat plate can be written as:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \mathcal{G} \frac{\partial^2 v_x}{\partial y^2}$$

where  $v_x$  and  $v_y$  are the fluid velocities parallel to and perpendicular to the flat plate respectively, and  $\mathcal{G}$  is the momentum diffusivity of the fluid.

Write down the boundary conditions for this flow situation given that the bulk fluid velocity far from the plate is  $V$ . [5]

A numerical solution to the boundary layer equations for fluid flowing over a flat plate is given by:

|         |   |       |       |       |       |       |      |
|---------|---|-------|-------|-------|-------|-------|------|
| $\eta$  | 0 | 0.1   | 0.2   | 0.3   | 1     | 2.5   | 5    |
| $v_x/V$ | 0 | 0.066 | 0.132 | 0.195 | 0.630 | 0.990 | 1.00 |

where  $\eta$  is a dimensionless variable given by:

$$\eta = y(V / 4\mathcal{G}x)^{0.5}$$

**CONTINUED**

where  $x$  is the distance along the plate from the leading edge,  $y$  is the distance perpendicular to the plate,  $V$  is the velocity of the bulk flow far from the plate and  $\mathcal{D}$  is the momentum diffusivity of the fluid.

Use this numerical solution to find:

- (i) An expression for the boundary layer thickness,  $\delta$ , and [5]
  - (ii) The local surface shear stress,  $\tau$ , [5]
- both as functions of distance  $x$  from the leading edge.

3.

A soluble gas component  $G$  is to be absorbed into a liquid containing a soluble component  $L$  with which it reacts instantaneously and irreversibly according to  $G + nL \rightarrow GL_n$ .

- (i) Derive describing equations for the *mass flux* and *critical concentration*. [6]
- (ii) Sketch how the rate of absorption of gas varies *quantitatively* with the concentration of  $L$  in the liquid within the range  $0 < [L] < 5 \text{ kmol m}^{-3}$ . [10]
- (iii) Calculate the maximum absorption enhancement factor. [2]
- (iv) Specify suitable types of contacting device over the range  $0 < [L] < 5 \text{ kmol m}^{-3}$ , with reasons. [2]

Data:

|  |  |
|--|--|
| Diffusion coefficients of $G$ and $L$ in the liquid phase    | $= 2.0 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$                          |
| Gas phase film mass transfer coefficient                     | $= 3.0 \times 10^{-3} \text{ kmol m}^{-2} \text{ s}^{-1} \text{ bar}^{-1}$ |
| Liquid phase mass transfer coefficient                       | $= 4.0 \times 10^{-4} \text{ m s}^{-1}$                                    |
| Partial pressure of $S$ in the bulk gas phase                | $= 0.10 \text{ bar}$   |
| Henry's Law coefficient for the solubility of gas in liquid. | $= 1.0 \times 10^2 \text{ bar m}^3 \text{ kmol}^{-1}$                      |
| Stoichiometric ratio, $n$                                    | $= 2$  |

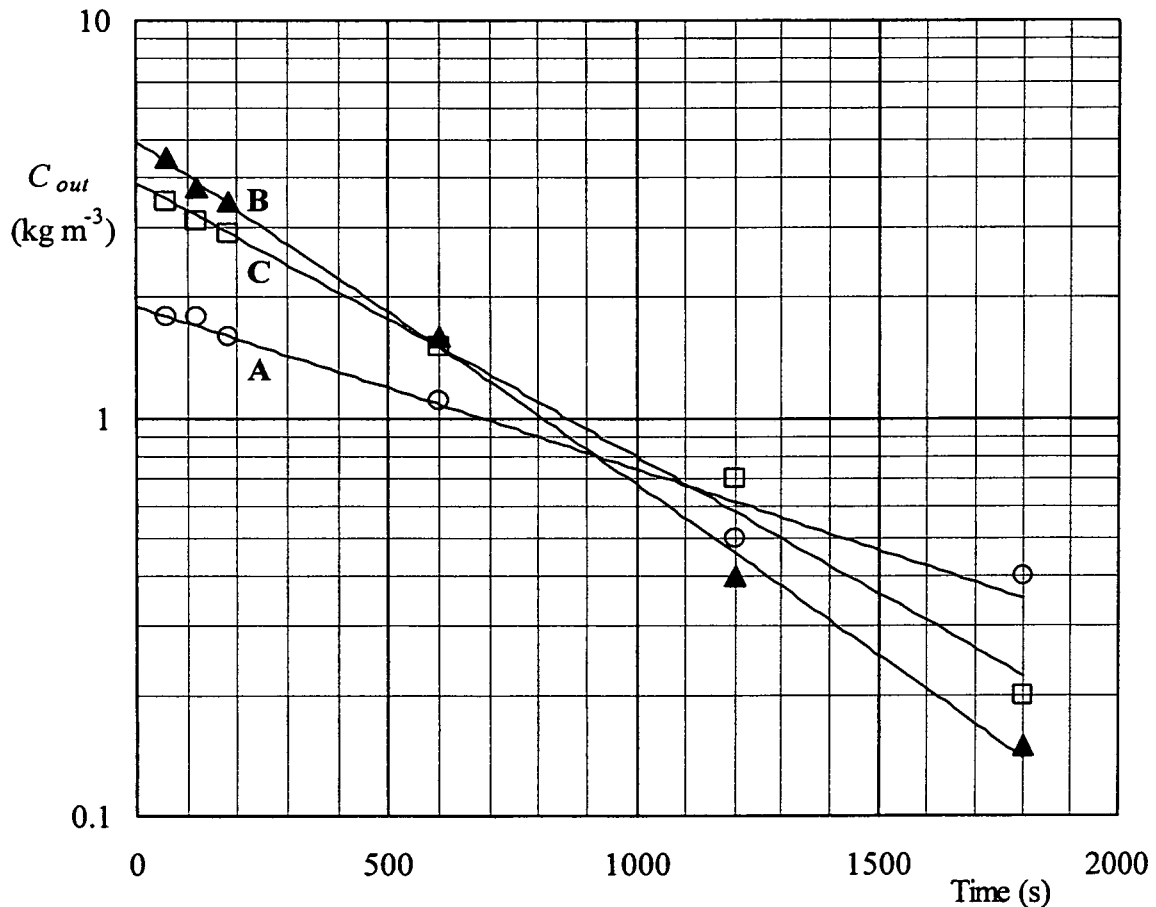
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4.

Discuss *briefly* the following expressions used when considering particle suspension in stirred tanks:

- (i) the just suspended condition; [2]
- (ii) complete particle suspension; [2]
- (iii) homogeneous particle suspension; and [2]
- (iv) what is the wash-out curve and how may it be used to characterise the suspension of solids in an agitated vessel? [2]

A series of washout tests were performed on one agitated vessel. Three different impellers, A, B and C, were tested and the results are shown in the following graph where exponential trend lines have been placed through the data.



In each test, 10 kg of powdered solid was added to the agitated vessel containing  $2 \text{ m}^3$  of liquid. Liquid flowed in and out of the vessel at a rate of  $4 \text{ L s}^{-1}$  and the concentration of solids in the outlet  $C_{out}$  was measured as a function of time.

Comment upon the performance of each of the three impellers in relation to the concepts of:

- (a) complete particle suspension; and
- (b) homogeneous particle suspension. [12]

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5.

Describe and discuss the importance of the *Metzner and Otto* method for estimating shear rates in stirred tanks.

[7]

Using the *Metzner and Otto* method, determine the Reynolds number for a power-law fluid (density  $935 \text{ kg m}^{-3}$ , fluid consistency  $285 \text{ Pa s}^{n-1}$  and flow behaviour index 0.27) agitated by two disc turbine impellers, each 0.4 m diameter, on a single shaft rotating at 80 r.p.m. in a 1 m diameter vessel. If the product of power number and Reynolds number for a single disc turbine in the laminar region is 50, estimate the total power required for agitation and the total torque on the impeller shaft. State clearly any assumptions that you have made.

[10]

What other factors would require consideration before the motor power could be specified?

[3]

6.

Outline the bases of:

(i) the Prandtl Mixing Length theory of turbulence, and

[5]

(ii) the Kolmogorov theory of turbulence.

[5]

A liquid flows through a smooth pipe of 200 mm internal diameter. If the mass flow rate of liquid is  $3750 \text{ kg min}^{-1}$ , estimate:-

(iii) the Prandtl scale of eddies on the pipe centre-line;

[2]

(iv) the Kolmogorov dissipation scale of turbulence; and

[6]

(v) the smallest scale of eddies present.

[2]

Data: liquid properties: density =  $1050 \text{ kg m}^{-3}$ , viscosity =  $1.65 \text{ mPa s}$ .  
 $c_f = 0.046 Re^{-0.2}$ .

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## Appendix

### Equations of Change

#### Rectangular co-ordinates ( $x, y, z$ ):

##### Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

##### Motion

###### $x$ -component

$$\rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left[ \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] + \rho g_x$$

###### $y$ -component

$$\rho \left( \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left[ \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] + \rho g_y$$

###### $z$ -component

$$\rho \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[ \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$

#### Cylindrical co-ordinates ( $r, \theta, z$ ):

##### Continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

##### Motion

###### $r$ -component

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \rho g_r$$

###### $\theta$ -component

$$\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] + \rho g_\theta$$

###### $z$ -component

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$

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