University of London

## EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-
B.Eng. Coll Dip M.Eng.

Chemical Eng E856: Transport Processes III

COURSE CODE : CENGE856

UNIT VALUE : 0.50

DATE : 24-MAY-04

TIME : $\mathbf{1 0 . 0 0}$

TIME ALLOWED : 3 Hours

## Answer FOUR questions.

Each question carries a total of 20 marks each, distributed as shown...
Only the first four answers will be marked.

1. A long vertical drill shaft of radius $R_{l}$ is sheathed by a stationary coaxial cylindrical vessel forming a well of radius $R_{2}$ containing a lubricating liquid. The shaft passes through the liquid free surface and rotates with angular velocity $\omega$.

Assuming that the liquid is Newtonian, derive expressions for the liquid velocity and pressure distributions respectively at a radial distance $r$ from the shaft axis.

Hence show that the height of the free liquid surface relative to a datum ( $h-h_{o}$ ) as a function of radial distance, $r$, is given by:

$$
h-h_{o}=\frac{K^{2}}{g}\left(2 R_{2}^{2} \ln \frac{R_{2}}{r}-\frac{R_{2}^{4}-r^{4}}{2 r^{2}}\right)
$$

where

$$
K=R_{1}^{2} \omega /\left(R_{2}^{2}-R_{1}^{2}\right)
$$

Continuity and Navier-Stokes equations:

$$
\frac{\partial \rho}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho \mathrm{rv}_{\mathrm{r}}\right)+\frac{1}{\mathrm{r}} \frac{\partial}{\partial \theta}\left(\rho v_{\theta}\right)+\frac{\partial}{\partial z}\left(\rho v_{z}\right)=0
$$

$r$-component

$$
\rho\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}-\frac{v_{\theta}{ }^{2}}{r}+v_{z} \frac{\partial v_{r}}{\partial z}\right)=-\frac{\partial \mathrm{p}}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(\mathrm{rv}_{r}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{r}}{\partial \theta^{2}}-\frac{2}{r^{2}} \frac{\partial v_{\theta}}{\partial \theta}+\frac{\partial^{2} v_{r}}{\partial z^{2}}\right]+\rho g_{r}
$$

$\theta$-component
$\rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{r} v_{\theta}}{r}+v_{z} \frac{\partial v_{\theta}}{\partial z}\right)=-\frac{1}{r} \frac{\partial \mathrm{p}}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{\theta}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{\theta}}{\partial \theta^{2}}+\frac{2}{r^{2}} \frac{\partial v_{r}}{\partial \theta}+\frac{\partial^{2} v_{\theta}}{\partial z^{2}}\right]+\rho g_{\theta}$
z-component

$$
\rho\left(\frac{\partial v_{z}}{\partial t}+v_{r} \frac{\partial v_{z}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{z}}{\partial \theta}+v_{z} \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial \mathrm{p}}{\partial z}+\mu\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_{z}}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{z}}{\partial \theta^{2}}+\frac{\partial^{2} v_{z}}{\partial z^{2}}\right]+\rho g_{z}
$$

2. (i) Convert the $x$-component of the equations of motion in rectangular co-ordinates provided below into dimensionless form by relating all the variables to suitable reference levels involving a characteristic length, $D$, and velocity, $V$. Explain the significance of the dimensionless groups involved and describe a practical application of this dimensionless form.
(ii) Fluid mixing in a full-scale gas phase reactor is to be investigated by constructing a small-scale model in a transparent material that will be operated with a liquid into which coloured tracer will be injected to show up the mixing patterns.

The kinematic viscosities ( $\mu / \rho$ ) for the gas and liquid are $1.5 \times 10^{-5}$ and $1 \times 10^{-6} \mathrm{~m}^{2} \mathrm{~s}^{-1}$, respectively. Suggest a linear scale factor for the model that will enable the water mixing observations to be related to the gas phase reactor performance.

Navier-Stokes equation for a Newtonian fluid with constant $\rho$ and $\mu$ :
$x$-component

$$
\rho\left(\frac{\partial v_{x}}{\partial t}+v_{x} \frac{\partial v_{x}}{\partial x}+v_{y} \frac{\partial v_{x}}{\partial y}+v_{z} \frac{\partial v_{x}}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^{2} v_{x}}{\partial x^{2}}+\frac{\partial^{2} v_{x}}{\partial y^{2}}+\frac{\partial^{2} v_{x}}{\partial z^{2}}\right]+\rho g_{x}
$$

3. Describe briefly how mass transfer can be important in determining the performance of chemical reactors.

Based on the film model, derive describing equations to predict the effect of a chemical reaction with irreversible second order reaction kinetics on: the rate of absorption of a gas dissolving into a liquid phase, the corresponding enhancement factor and the critical concentration, respectively.

Component $G$ is absorbed from a gas stream at a partial pressure of $2.0 \times 10^{-1}$ bar into a liquid containing $L$ where it reacts according to:

$$
G+\mathrm{n} L=G L_{\mathrm{n}}
$$

The reaction is known to be irreversible and practically instantaneous. Calculate, and illustrate graphically, how the rate of absorption of the gas changes as the concentration of $L$ increases from zero to $1.0 \mathrm{kmol} \mathrm{m}^{-3}$. Also estimate the effect of a further doubling of the concentration of $L$ to $2.0 \mathrm{kmol} \mathrm{m}^{-3}$.

## Data:

Diffusion coefficients of both $G$ and $L$ in the liquid phase $=6 \times 10^{-10} \mathrm{~m}^{2} \mathrm{~s}^{-1}$
Gas phase mass transfer coefficient

$$
=4 \times 10^{-5} \mathrm{kmol} \mathrm{~m}^{-2} \mathrm{~s}^{-1} \mathrm{bar}^{-1}
$$

Liquid phase mass transfer coefficient $=3 \times 10^{-5} \mathrm{~ms}^{-1}$ Henry's Law coefficient for the solubility of $G$ in $L$.
$=1 \times 10^{-3}$ bar m $\mathrm{mmol}^{-1}$
Stoichiometric ratio for the liquid phase reaction

$$
=2
$$

## CONTINUED

4. 6 tonnes $h^{-1}$ of ideal gas (molecular weight $=44$, pressure $=28$ bar, temperature $=$ $170{ }^{\circ} \mathrm{C}$ and viscosity $=0.45 \mathrm{mPa} \mathrm{s}$ ) flows inside a smooth straight circular pipe of 0.15 m internal diameter. Given that $\frac{1}{\sqrt{c_{f}}}=4.0 \log _{10}\left(\operatorname{Re} \sqrt{c_{f}}\right)-0.40$ and that the velocity profile in the turbulent core is given by the equation: $v^{+}=2.5 \ln y^{+}+5.5$, where $v^{+}=v / v^{*}, y^{+}=y v^{*} \rho / \mu, v^{*}$ is the friction, or shear, velocity, $v$ is the velocity at a distance $y$ from the pipe wall, $\rho$ is the fluid density and $\mu$ the fluid viscosity, estimate:
(i) the velocity of liquid 0.05 m from the pipe wall;
(ii) the velocity at the laminar sub-layer/buffer region interface;
(iii) the thickness of the laminar sub-layer; and
(iv) the Prandtl mixing length at the pipe centre-line.
5. A baffled cylindrical vessel is filled to a height equal to the vessel diameter with a low viscosity Newtonian liquid. This liquid is agitated using a centrally-mounted, standard Rushton disk-turbine with clearance from the bottom of the vessel equal to $40 \%$ of the vessel diameter. Discuss, with the aid of diagrams, the effect of gas addition on the power requirements of the impeller when air is injected into the liquid through a single sparger placed centrally under the impeller.

Air is injected at a rate of 2 VVM (volume of gas per unit volume of vessel per minute) into the vessel containing $1.2 \mathrm{~m}^{3}$ of a Newtonian liquid, density $1120 \mathrm{~kg} \mathrm{~m}^{-3}$, and viscosity 0.015 Pa s . The vessel is equipped with two impellers on the same shaft each having diameter, $D$, equal to 0.2 m , with a separation of $2 D$. The impellers operate at a rotational speed, $N$, of 150 rpm . Using the correlation below, predict the power input, $P_{g}$, under aerated conditions.

$$
\begin{equation*}
P_{g}=C\left(\frac{P^{2} N D^{3}}{Q^{0.56}}\right)^{0.45} \tag{6}
\end{equation*}
$$

where $C$ is a constant which is equal to 0.72 when SI units are used, $Q$ is the gassing rate and $P$ is the impeller power input for the ungassed condition. Ungassed power numbers for single impellers may be estimated using the relationships:

$$
\begin{array}{ll}
P o=80 R e^{-1} & \text { in the laminar regime, and } \\
P o=6 & \text { in the turbulent regime } .
\end{array}
$$

What do you expect to be the uncertainty in your prediction?
6. Define the "generalised fluid" in non-Newtonian fluid mechanics and state how it differs from the "power-law" fluid.

The following data on frictional pressure drop, $\Delta p_{f}$, versus volumetric flow rate, $Q$, of a non-Newtonian slurry, density $=4500 \mathrm{~kg} \mathrm{~m}^{-3}$, was obtained using a capillary tube viscometer with an inner diameter of 1.5 mm and 300 mm long.

| $\Delta p_{f}$ | $(\mathrm{kPa})$ | 30 | 60 | 120 | 240 | 480 |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
| $Q$ | $\left(\mathrm{~cm}^{3} \mathrm{~s}^{-1}\right)$ | 0.017 | 0.044 | 0.14 | 0.35 | 1.2 |

$800 \mathrm{~kg} \mathrm{~s}^{-1}$ of this slurry flows along a 290 m length of 0.25 m internal diameter pipeline. Using Dodge and Metzner's friction factor $c_{f}$ versus generalised Reynolds number $R e^{*}$ below estimate the frictional pressure drop.


Fanning friction factor chart for generalised fluids

Note: Log-log paper is provided (attached). Insert it into the Answer Book opposite your answer.

