

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B. Eng.

M. Eng.

Chemical Eng E856: Transport Processes III

COURSE CODE : **CENGE856**

UNIT VALUE : **0.50**

DATE : **30-APR-02**

TIME : **10.00**

TIME ALLOWED : **3 hours**

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TURN OVER

Answer **FOUR** questions only.

Each question carries a total of 20 marks distributed as shown [].

The Equations of Change are appended.

1. A long horizontal pipe of circular cross-section and radius R is filled with a Newtonian liquid of viscosity μ and density ρ . A continuous wire, radius xR ($x < 1$) is drawn along the pipe axis at a steady velocity V .

Using the continuity and Navier-Stokes equations in the appended equations of change, derive an equation describing the velocity profile in the liquid. Neglect any end effects. [10]

If the pipe is 20 m long, $\mu = 10^{-1}$ Pa s, $x = 2 \times 10^{-1}$ and $V = 2$ m s⁻¹, calculate the force required to draw the wire through the liquid. [10]

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2. Describe, with the aid of a sketch, the phenomenon of the boundary layer with reference to the two dimensional flow of a Newtonian fluid along a horizontal, flat plate.

[5]

The differential describing equations describing the flow within the boundary layer along a flat plate can be written as:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \mathcal{G} \frac{\partial^2 v_x}{\partial y^2}$$

where v_x and v_y are the fluid velocities parallel to and perpendicular to the flat plate respectively, and \mathcal{G} is the momentum diffusivity of the fluid.

Write down the boundary conditions for this flow situation given that the bulk fluid velocity far from the plate is V .

[5]

A numerical solution to the boundary layer equations for fluid flowing over a flat plate is given by:

| | | | | | | | |
|---------|---|--------|-------|-------|-------|-------|------|
| η | 0 | 0.1 | 0.2 | 0.3 | 1 | 2.5 | 5 |
| v_x/V | 0 | 0.0665 | 0.133 | 0.195 | 0.630 | 0.990 | 1.00 |

where η is a dimensionless variable given by:

$$\eta = y \left(\frac{V}{4\mathcal{G}x} \right)^{0.5}$$

where x is the distance along the plate from the leading edge, y is the distance perpendicular to the plate, V is the velocity of the bulk flow far from the plate and \mathcal{G} is the momentum diffusivity of the fluid.

Use this numerical solution to find

- (i) An expression for the boundary layer thickness, δ , and

[5]

- (ii) The local surface shear stress, τ_0 ,

[5]

both as functions of distance x from the leading edge.

TURN OVER

3. A soluble gas component S is to be absorbed into a liquid containing soluble component R which it reacts instantaneously and irreversibly according to $S + qR \rightarrow SR_q$.

- (i) Derive describing equations for the *mass flux* and *critical concentration*. [6]
- (ii) Sketch how the rate of absorption of gas varies *quantitatively* with the concentration of R in the liquid within the range $0 < [R] < 10 \text{ kmol m}^{-3}$. [10]
- (iii) Calculate the maximum absorption enhancement factor. [2]
- (iv) Specify suitable types of contacting device over the range $0 < [R] < 10 \text{ kmol m}^{-3}$, with reasons. [2]

Data:

| | |
|---|--|
| Diffusion coefficients of S and R in the liquid phase | $= 1.0 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ |
| Gas phase film mass transfer coefficient | $= 2.0 \times 10^{-3} \text{ kmol m}^{-2} \text{ s}^{-1} \text{ bar}^{-1}$ |
| Liquid phase mass transfer coefficient | $= 1.0 \times 10^{-4} \text{ m s}^{-1}$ |
| Partial pressure of S in the bulk gas phase | $= 0.15 \text{ bar}$ |
| Henry's Law coefficient for the solubility of gas in liquid | $= 1.2 \times 10^2 \text{ bar m}^3 \text{ kmol}^{-1}$ |
| Stoichiometric ratio | $= 2$ |

TURN OVER

4. Show that the volumetric flow rate Q through a pipe of internal radius R for a Bingham plastic, $\dot{\gamma} = \frac{\tau_y - \tau}{\mu_p}$, are related to the wall shear stress τ_w by

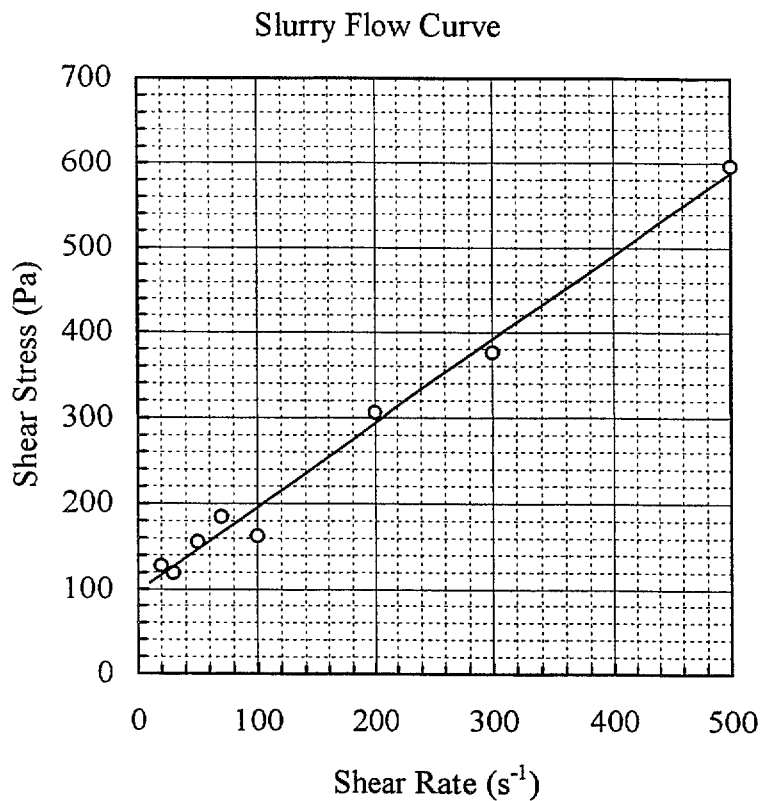
$$\frac{4Q}{\pi R^3} = \frac{\tau_w}{\mu_p} \left[1 - \frac{4}{3} \left(\frac{\tau_y}{\tau_w} \right) + \frac{1}{3} \left(\frac{\tau_y}{\tau_w} \right)^4 \right]$$

where $\dot{\gamma}$ is the shear rate, τ the shear stress, τ_y the fluid yield stress and μ_p the plastic viscosity. [10]

A slurry is to be pumped through a 130 m long, straight horizontal, 180 mm diameter pipe. Calculate:

- (i) the pressure drop required to just start the slurry flowing; [2]
- (ii) the mass flowrate when a pressure difference of 6 bar is applied across the pipe length; and [6]
- (iii) the fraction of the pipe cross-sectional area occupied by the “solid” central plug at the flowrate given in (ii). [2]

Data: Slurry properties: density 1250 kg m^{-3} ; “flow curve” below.



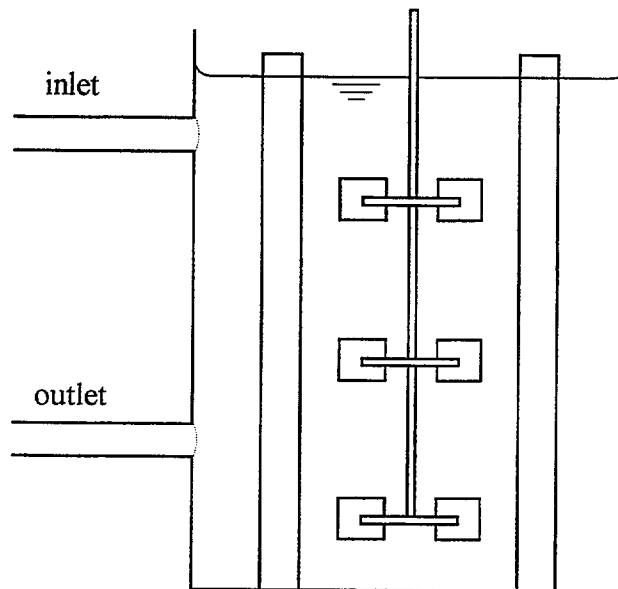
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5. It is proposed to scale-up a continuous stirred tank reactor to maintain the same product yield per unit volume per unit time. To do this, it has been recommended that scale-up be at constant residence time distribution and at constant mean residence time.

- (i) Derive an appropriate scale-up criterion to achieve this under geometric similarity.

[6]

A pilot plant continuous stirred tank reactor (illustrated below) has an internal diameter of 300 mm and is filled to a depth of 500 mm. It is fitted with three Rushton (disk) turbines, each with a diameter one third of that of the tank, mounted one half of a tank diameter apart on a single impeller shaft. This impeller shaft rotates at 600 rpm and is centrally mounted in the vessel, which is fully baffled.



Pilot scale stirred tank reactor

Experiments indicate that a mean residence time of 15 minutes achieves the required product yield.

- (ii) Calculate the mass flowrate of reactants into the pilot scale vessel. [2]
- (iii) Using the scale-up criterion derived in (i) above, calculate the diameter of a geometrically similar production-scale reactor to handle 2 kg s^{-1} feed. [3]
- (iv) What are the power and torque required from a motor drive to the impellers for your production-scale reactor sized in (iii)? [6]
- (v) Comment upon any drawbacks of this scale-up criterion. [3]

Data: Reaction mixture density = 1150 kg m^{-3} , viscosity = $0.56 \times 10^{-3} \text{ Pa s}$.
The power number for a single Rushton (disk) turbine is 0.5 .

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6. Outline the bases of:

(i) the *Prandtl Mixing Length* theory of turbulence, and [5]

(ii) the *Kolmogorov* theory of turbulence. [5]

An ideal gas at 5 bar and 270 °C flows through a pipe 600 mm internal diameter. If the mass flow rate of gas is 50 kg min⁻¹, estimate:-

(iii) the Prandtl scale of eddies on the pipe centre-line; [2]

(iv) the Kolmogorov dissipation scale of turbulence; and [6]

(v) the smallest scale of eddies present. [2]

Data: gas properties: molecular mass = 17, viscosity = 1.9×10^{-5} Pa s.
 $c_f = 0.079Re^{-0.25}$. $R = 8.314$ kJ kg⁻¹ K⁻¹.

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Appendix

Equations of Change

Rectangular co-ordinates (x, y, z):

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Motion

x -component

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] + \rho g_x$$

y -component

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] + \rho g_y$$

z -component

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$

Cylindrical co-ordinates (r, θ, z):

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Motion

r -component

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \rho g_r$$

θ -component

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] + \rho g_\theta$$

z -component

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$

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