

Answer **FOUR QUESTIONS**, two from **PART A** (questions 1, 2, and 3) and two from **PART B** (questions 4, 5 and 6).
ALL questions carry a total of **25 MARKS** each, distributed as shown []

PART A

1. a) What is meant by a mathematical model of a physical process? [2]
- b) Why do we need to develop the mathematical model of a process we wish to control? [3]
- c) What are state variables and state equations? What are they used for? What are the most common state variables for chemical processes? [6]
- d) Consider the system shown in figure 1. For tank 1, steam is injected directly into the liquid water. Water vapour is produced in the second tank. The effluent flowrates are proportional to the liquid static pressure that causes their flow. λ_{v1} and λ_{v3} are the heats of condensation at the pressure in the first tank and of vaporisation in the second tank, respectively, and ΔH_{v2} is the heat of condensation in the steam coil.
 - i) Identify the dependent state variables of the system.
 - ii) Determine the balances that are required in order to develop the state equations.
 - iii) Develop the state model, with appropriate assumptions, that describes the dynamic behaviour of the system. [12]
- e) How many degrees of freedom are there in the state model developed in (d)? Which variables need to be specified so that the model can be solved completely? [2]

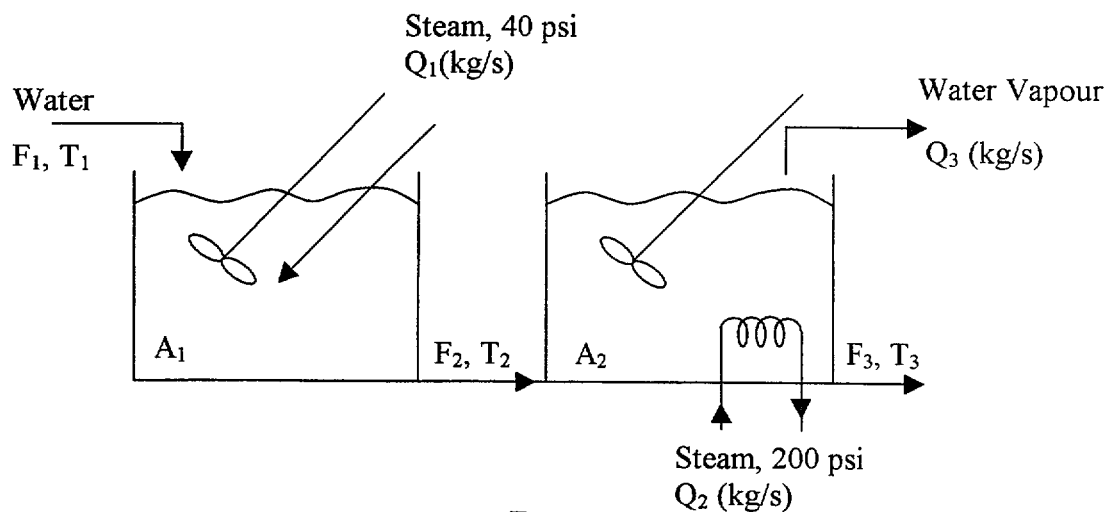


Figure 1

TURN OVER

2. a) What is a second-order system? Write the differential equation describing second-order behaviour in the time domain and give its transfer function. From which physical situations can such behaviour arise? [5]
- b) Discuss, with the aid of a diagram, the overdamped response to a unit step input of a second-order system, identifying its distinguishing characteristics. Where do such responses most commonly occur in chemical processes? Compare the response, using the same diagram, with that of a first-order system to the same input. [6]
- c) Draw the block diagram for two non-interacting first-order capacities, showing the input and output variables and the transfer function for each capacity. [3]
- d) By using the diagram drawn in (c), show that for two non-interacting first order capacities where there is a unit step input, the output response is given by:

$$y(t) = K'_p \left[1 + \frac{1}{\tau_{p2} - \tau_{p1}} \left(\tau_{p1} e^{-t/\tau_{p1}} - \tau_{p2} e^{-t/\tau_{p2}} \right) \right]$$

where $K'_p = K_{p1}K_{p2}$

[11]

TURN OVER

3. An unstable first-order process with transfer function:

$$G_p(s) = \frac{2}{s-4}$$

is controlled by a proportional controller. Assuming $G_m(s) = G_f(s) = 1$ and

$$G_d(s) = \frac{1}{s+1}$$

- a) Determine the characteristic equation of the closed-loop system. [2]
- b) Using the Routh-Hurwitz criterion, determine the range of values of proportional gain which would yield stable closed-loop responses. [2]
- c) What is the major advantage of the Routh-Hurwitz criterion for examining the stability of a system? [2]
- d) Find the value of proportional gain for which the closed-loop pole is zero. Using this value, compute the closed-loop response to a unit step change in the set point. Is this response stable or unstable and why? [6]
- e) Integral action is added to the controller. Show that:
 - i) for all integral times, τ_i , the stability criterion remains unchanged from that found in (b)
 - ii) the addition of integral action eliminates offset when there is a unit step change in the set point. [13]

General formula for the Routh array:

Row	1	a_0	a_2	a_4	a_6
	2	a_1	a_3	a_5	a_7
	3	A_1	A_2	A_3
	4	B_1	B_2	B_3

where: $A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$, $A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$, $A_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$

$B_1 = \frac{A_1 a_3 - a_1 A_2}{A_1}$, $B_2 = \frac{A_1 a_5 - a_1 A_3}{A_1}$

TURN OVER

PART B

4. A process controlled by a feedback loop is known to exhibit first order dynamics and contains a time delay. The process gain is K_p , the time constant is τ_p and the dead time τ_d . The dynamics for the valve and measurement can be neglected. The controller is a P controller with gain K_c . A disturbance $d(s)$ with transfer function $G_d(s)$ is present.
- a) Set up the block diagram for the closed loop system. [2]
 - b) Find the transfer function relating set point and load changes with the output variable. [2]
 - c) Find expressions for the amplitude ratio and the phase shift for the open loop process. [4]
- Assuming no disturbance and keeping the controller gain at $K_c = 1$, an experiment is performed to determine the time constant τ_p and the time delay τ_d of the process. It is found that the process gain K_p is 2 and the crossover frequency for the process is $\omega_{co} = 20$ rad/min.
- d) At the crossover frequency, the amplitude ratio is equal to 0.0198. What is the value of the process time constant τ_p ? [6]
 - e) What is the value of the process deadtime τ_d ? [6]
 - f) With the values for the time constant τ_p and the time delay τ_d of the process found above, sketch the open loop response for the process (without the controller) to a unit step in the input. Indicate on your sketch the process gain K_p , the time constant τ_p and the dead time τ_d . [5]

TURN OVER

5. a) Give examples of where time delays may occur in chemical processes and explain briefly why time delays can cause unsatisfactory closed loop responses for conventional feedback controllers. [4]
- b) Set up the block diagram for a closed loop system where a standard feedback controller, together with a deadtime compensator, is used to control a process with transfer function $G_p(s) = G(s)e^{-\tau_d s}$. Assume no disturbance $d(s) = 0$ and no dynamics in the valve and measurement, i.e. $G_f = G_m = 1$. Find the transfer function for the deadtime compensator. [6]
- c) What are the benefits of a deadtime compensator? What are the potential problems with using dead time compensators? [4]
- d) What is inverse response? Give a chemical engineering example of a process which exhibits inverse response and explain briefly how the inverse response can be observed. [5]
- e) Consider the process given in Figure 2 of two first order systems combined. Sketch the open loop response of the overall process when:
- $\tau_1 / \tau_2 > K_1 / K_2 > 1$
 - $\tau_1 / \tau_2 = K_1 / K_2 > 1$
 - $\tau_1 / \tau_2 < 1$ and $K_1 / K_2 > 1$
 - $\tau_1 / \tau_2 > 1$ and $K_1 / K_2 < 1$
- [6]

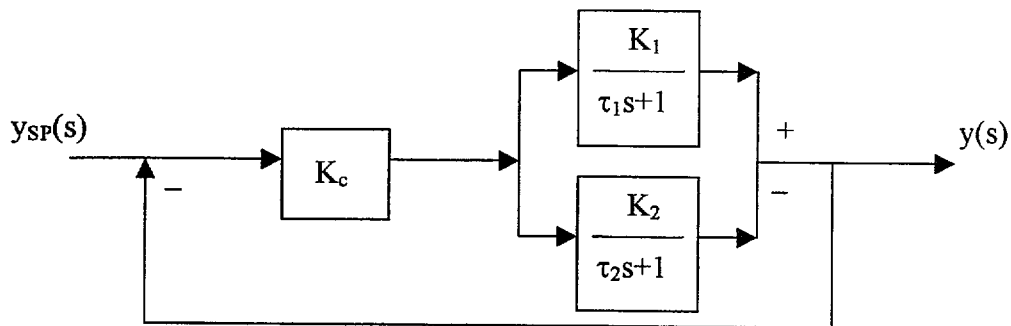


Figure 2. Two first order systems.

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6. The process in Figure 3 is to be controlled using two feedback control loops.

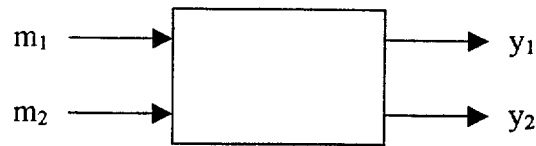


Figure 3. Multivariable system.

a) Explain briefly in words what interaction in a multivariable process means and why this may cause problems for the control of the process. [4]

In order to determine the controller pairing, two experiments are performed:

- i) With both loops open, m_2 is kept constant and a unit step in m_1 is introduced and the response in y_1 is recorded. The response is given to the left in Figure 4.
- ii) With y_2 assumed perfectly controlled using m_2 , a unit step in m_1 is introduced and the response in y_1 is recorded. The response is given to the right in Figure 4.

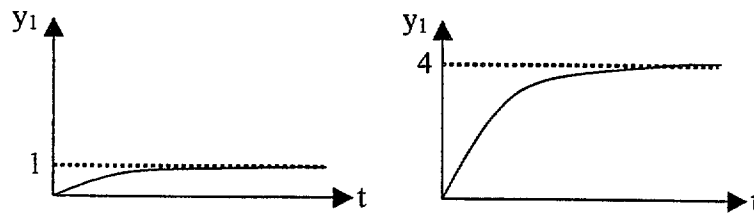


Figure 4. Step responses for multivariable system.

Left: m_2 constant, Right: y_2 constant.

b) Calculate the Relative Gain Array for the process from the step responses in Figure 4. [5]

c) Suggest a controller pairing for the process and explain your choice. [4]

The transfer function for a process similar to the one given in Figure 3 is given by:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} m_1(s) \\ m_2(s) \end{bmatrix} = \begin{bmatrix} \frac{2e^{-0.1s}}{10s+1} & \frac{5e^{-7s}}{75s+1} \\ \frac{3e^{-5s}}{100s+1} & \frac{4e^{-0.15s}}{3s+1} \end{bmatrix} \cdot \begin{bmatrix} m_1(s) \\ m_2(s) \end{bmatrix}$$

d) Comment on the dynamics of the proposed controller pairing if the pairing from c) was to be used for this process. [4]

e) Suggest possible ways of improving the control of the process beyond having just two feedback control loops. [4]

f) The control loops with the pairing found in c) are to be implemented using digital computer control. How often would you sample the loop controlling y_1 ? How often would you sample the loop controlling y_2 ? [4]

END OF PAPER