## UNIVERSITY COLLEGE LONDON

University of London

## **EXAMINATION FOR INTERNAL STUDENTS**

For The Following Qualification:-

· • • •

M.Sc.

**M9: Transport Processes** 

COURSE CODE	:	CENG00M9
DATE	:	24-MAY-04
ТІМЕ	:	10.00
TIME ALLOWED	:	3 Hours

04-N0012-3-30 © 2004 University College London

**TURN OVER** 

. .....

Answer FOUR other questions from the rest of the paper.

Each question carries a total of 20 marks each, distributed as shown...

Only the first four answers will be marked.

1. A long vertical drill shaft of radius  $R_1$  is sheathed by a stationary coaxial cylindrical vessel forming a well of radius  $R_2$  and contains a lubricating liquid. The shaft passes through the liquid free surface and rotates with angular velocity  $\omega$ .

Assuming that the liquid is Newtonian, derive expressions for the liquid velocity and pressure distributions respectively at a radial distance r from the shaft axis. [16]

Hence show that the height of the free liquid surface relative to a datum  $(h - h_o)$  as a function of radial distance, r, is given by:

$$h - h_o = \frac{K^2}{g} \left( 2R_2^2 \ln \frac{R_2}{r} - \frac{R_2^4 - r^4}{2r^2} \right)$$

where

$$K = R_1^2 \omega / (R_2^2 - R_1^2).$$

Continuity and Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

. . . . . .

r-component

$$\rho\left(\frac{\partial \mathbf{v}_{\mathbf{r}}}{\partial t} + \mathbf{v}_{\mathbf{r}}\frac{\partial \mathbf{v}_{\mathbf{r}}}{\partial \mathbf{r}} + \frac{\mathbf{v}_{\theta}}{\mathbf{r}}\frac{\partial \mathbf{v}_{\mathbf{r}}}{\partial \theta} - \frac{\mathbf{v}_{\theta}^{2}}{\mathbf{r}} + \mathbf{v}_{z}\frac{\partial \mathbf{v}_{\mathbf{r}}}{\partial z}\right) = -\frac{\partial \mathbf{p}}{\partial \mathbf{r}} + \mu\left[\frac{\partial}{\partial \mathbf{r}}\left(\frac{1}{\mathbf{r}}\frac{\partial}{\partial \mathbf{r}}(\mathbf{r}\mathbf{v}_{\mathbf{r}})\right) + \frac{1}{\mathbf{r}^{2}}\frac{\partial^{2}\mathbf{v}_{\mathbf{r}}}{\partial \theta^{2}} - \frac{2}{\mathbf{r}^{2}}\frac{\partial \mathbf{v}_{\theta}}{\partial \theta} + \frac{\partial^{2}\mathbf{v}_{\mathbf{r}}}{\partial z^{2}}\right] + \rho \mathbf{g}_{\mathbf{r}}$$

 $\theta$ -component

$$\rho\left(\frac{\partial \mathbf{v}_{\theta}}{\partial t} + \mathbf{v}_{r}\frac{\partial \mathbf{v}_{\theta}}{\partial r} + \frac{\mathbf{v}_{\theta}}{r}\frac{\partial \mathbf{v}_{\theta}}{\partial \theta} + \frac{\mathbf{v}_{r}\mathbf{v}_{\theta}}{r} + \mathbf{v}_{z}\frac{\partial \mathbf{v}_{\theta}}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(\mathbf{r}\mathbf{v}_{\theta})\right) + \frac{1}{r^{2}}\frac{\partial^{2}\mathbf{v}_{\theta}}{\partial \theta^{2}} + \frac{2}{r^{2}}\frac{\partial \mathbf{v}_{r}}{\partial \theta} + \frac{\partial^{2}\mathbf{v}_{\theta}}{\partial z^{2}}\right] + \rho \mathbf{g}_{\theta}$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_{\theta}}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

## **TURN OVER**

[4]

[]

- 2. (i) Convert the x-component of the equations of motion in rectangular co-ordinates provided below into dimensionless form by relating all the variables to suitable reference levels involving a characteristic length, D, and velocity, V. Explain the significance of the dimensionless groups involved and describe a practical application of this dimensionless form.
  - (ii) Fluid mixing in a full-scale gas phase reactor is to be investigated by constructing a small-scale model in a transparent material that will be operated with a liquid into which coloured tracer will be injected to show up the mixing patterns.

The kinematic viscosities  $(\mu/\rho)$  for the gas and liquid are  $1.5 \times 10^{-5}$  and  $1 \times 10^{-6} \text{ m}^2 \text{s}^{-1}$ , respectively. Suggest a linear scale factor for the model that will enable the water mixing observations to be related to the gas phase reactor performance.

1

[10]

[10]

[2]

[8]

*Navier-Stokes* equation for a Newtonian fluid with constant  $\rho$  and  $\mu$ :

*x-component* 

$$\rho\left(\frac{\partial \mathbf{v}_{\mathbf{x}}}{\partial t} + \mathbf{v}_{\mathbf{x}}\frac{\partial \mathbf{v}_{\mathbf{x}}}{\partial \mathbf{x}} + \mathbf{v}_{\mathbf{y}}\frac{\partial \mathbf{v}_{\mathbf{x}}}{\partial \mathbf{y}} + \mathbf{v}_{\mathbf{z}}\frac{\partial \mathbf{v}_{\mathbf{x}}}{\partial \mathbf{z}}\right) = -\frac{\partial \mathbf{p}}{\partial \mathbf{x}} + \mu\left[\frac{\partial^{2}\mathbf{v}_{\mathbf{x}}}{\partial \mathbf{x}^{2}} + \frac{\partial^{2}\mathbf{v}_{\mathbf{x}}}{\partial \mathbf{y}^{2}} + \frac{\partial^{2}\mathbf{v}_{\mathbf{x}}}{\partial \mathbf{z}^{2}}\right] + \rho \mathbf{g}_{\mathbf{x}}$$

3. Describe *briefly* how mass transfer can be important in determining the performance of chemical reactors.

Based on the film model, derive describing equations to predict the effect on the rate of absorption into a liquid phase of a dissolving gas undergoing a chemical reaction with irreversible second order reaction kinetics, the enhancement factor and critical concentration, respectively.

Component G is absorbed from a gas stream at a partial pressure of  $2.0 \times 10^{-1}$  bar into a liquid containing L where it reacts according to:

$$G + nL = GL_n$$

The reaction is known to be irreversible and practically instantaneous. Calculate, and illustrate graphically, how the rate of absorption of the gas changes as the concentration of L varies from zero to 1.0 kmol m<sup>-3</sup>. Also estimate the effect of a further doubling of the concentration of L. [10]

Data:

Diffusion coefficients of both G and L in the liquid phase
$$= 6 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$$
Gas phase mass transfer coefficient $= 4 \times 10^{-5} \text{ kmol m}^2 \text{ s}^{-1} \text{ bar}^{-1}$ Liquid phase mass transfer coefficient $= 3 \times 10^{-5} \text{ ms}^{-1}$ Henry's Law coefficient for the solubility of G in L. $= 1 \times 10^{-3} \text{ bar m}^3 \text{ kmol}^{-1}$ Stoichiometric ratio for the liquid phase reaction $= 2$ 

CONTINUED

3

4. 6 tonnes h<sup>-1</sup> of ideal gas (molecular weight = 44, pressure = 28 bar, temperature = 170 °C and viscosity = 0.45 mPa s) flows inside a smooth straight circular pipe of 0.15 m internal diameter. Given that  $\frac{1}{\sqrt{c_f}} = 4.0 \log_{10} \left( Re \sqrt{c_f} \right) - 0.40$  and that the

velocity profile in the turbulent core is given by the equation:  $v^+ = 2.5 \ln y^+ + 5.5$ , where  $v^+ = v/v^*$ ,  $y^+ = yv^* \rho/\mu$ ,  $v^*$  is the friction, or shear, velocity, v is the

velocity at a distance y from the pipe wall,  $\rho$  is the fluid density and  $\mu$  the fluid viscosity, estimate:

- (i) the velocity of liquid 0.05 m from the pipe wall;
  (ii) the velocity at the laminar sub-layer/buffer region interface;
  [5]
- (iii) the thickness of the laminar sub-layer; and [5]
- (iv) the Prandtl mixing length at the pipe centre-line.
- 5. Discuss, with the aid of diagrams, the effect of gas addition on the power requirements of a standard Rushton, disk-turbine, impeller in a vessel containing a low viscosity Newtonian liquid in which air is injected into the liquid through a single sparger placed centrally under the impeller where the impeller clearance from the bottom of the vessel is 40% of the vessel diameter. [12]

Air is injected at a rate of 2 VVM (volume of gas per unit volume of vessel per minute) into the vessel containing  $1.2 \text{ m}^3$  of a Newtonian liquid, density 1120 kg m<sup>-3</sup>, and viscosity 0.015 Pa s. The vessel is equipped with two impellers on the same shaft each having diameter, D, equal to 0.2 m, with a separation of 2D. The impellers operate at a rotational speed, N, of 150 rpm. Using the correlation below, predict the power input,  $P_g$ , under aerated conditions. [6]

$$P_{g} = C \left(\frac{P^2 N D^3}{Q^{0.56}}\right)^{0.4}$$

where C is a constant which is equal to 0.72 when SI units are used, Q is the gassing rate and P is the impeller power input for the ungassed condition. Ungassed power numbers for single impellers may be estimated using the relationships:

$Po = 80 Re^{-1}$	in the laminar regime, and
<i>Po</i> = 6	in the turbulent regime.

What do you expect to be the uncertainty in your prediction?

## **TURN OVER**

[2]

[5]

CENGM9

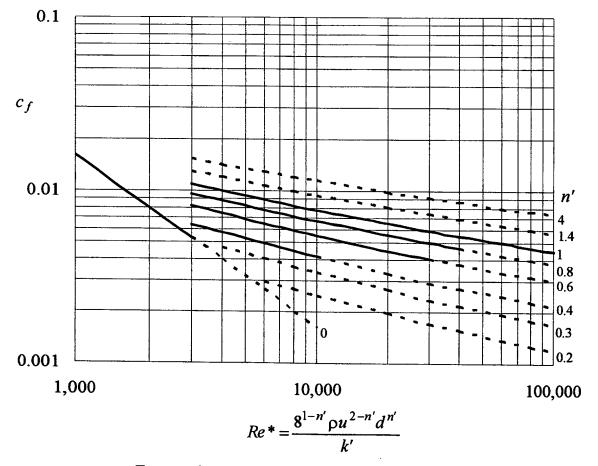
١

6. Define the "generalised fluid" in non-Newtonian fluid mechanics and state how it differs from the "power-law" fluid.

The following data on frictional pressure drop,  $\Delta p_f$ , versus volumetric flow rate, Q, of a non-Newtonian slurry, density = 4500 kg m<sup>-3</sup>, was obtained using a capillary tube viscometer with an inner diameter of 1.5 mm and 300 mm long.

$\Delta p_f$	(kPa)	30	60	120	240	480
Q	$(cm^3 s^{-1})$	0.017	0.044	0.14	0.35	1.2

800 kg s<sup>-1</sup> of this slurry flows along a 290 m length of 0.25 m internal diameter pipeline. Using Dodge and Metzner's friction factor  $c_f$  versus generalised Reynolds number  $Re^*$  below estimate the frictional pressure drop. [15]



Fanning friction factor chart for generalised fluids

Note: Log-log paper is provided (attached). Insert it into your Answer Book opposite your answer.

**END OF PAPER** 

5

[5]

t