UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sc.

M6: Advanced Process Engineering

COURSE CODE	:	**CENG00M6**
DATE	:	**09-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**3 Hours**

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*Answer **FOUR** questions. Each question carries a total of 25 marks, distributed as shown [ ]. Only the FIRST **FOUR** ANSWERS will be marked.*

1.

a) State the Kuhn Tucker conditions for a constrained minimum of a function with constraints. Define all your variables. [5]

b) Form the Lagrangian for the following problem and find the minimum:

Min $x_1^2 + x_2^2 + 10x_1 + 20x_2 + 25$

Subject to $x_1 + x_2 = 0$ [7]

c) What is a Quadratic Programming problem, how is it solved, and what is its role in the Successive Quadratic Programming method for solving nonlinear programming problems? [7]

d) Reduced Gradient methods sometimes take steps that lead to positions outside the feasible region. Why is this so and how is feasibility restored? [6]

2.

a) Give three objective functions that are useful in Process Design and explain how they are determined and when they might be used. [3]

b) Plot the contours for the following function

$$(x_1 - 6)^2 + (x_2 - 2)^2 + 10$$ [2]

c) What is the difference between a strong and a weak minimum of a function? [2]

d) Give three ways of obtaining Hessian matrices in gradient methods for solving unconstrained optimisation methods and explain why and under which circumstances you might choose each one. [6]

e) What are the properties of the Hessian matrices that are maintained by Quasi Newton methods? [3]

f) Explain how a step is generated in the Levenberg Marquardt method. [2]

g) Perform one iteration of a gradient based method without line search for the following problem starting from the point $[1\ \ 1]^T$

$$\text{Minimise } (x_1 - 6)^2 + (x_2 - 2)^2 + 10$$ [7]

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3.

a) Let Y be a binary variable to denote the existence of a reactor. Formulate the following statement as a set of constraints that are *linear* in Y:

"A reactor with volume V has a fixed cost A and a variable cost $B \cdot V^{0.6}$. The volume of the reactor should be larger than L and smaller than U if the reactor is selected, otherwise it should be forced to the value of 0." [8]

b) A European company considers building M warehouses studying K alternative locations in Europe so as to serve N customer regions. Each customer region i can be reached (served) by a potential warehouse at location k within T_{ik} minutes. Note that all response times (T_{ik}) are known parameters. The objective is to select the best M locations to build warehouses in order to minimise the maximum response from any selected location to the customer regions to be served. Assume that each customer region should be allocated to exactly one warehouse. Formulate the above problem as a mixed integer linear programming model. [17]

4.

a) Formulate the following logical implication by a mixed integer linear set of constraints by using 0-1 variables:

$$If \left(\sum_i A_i X_i \le B\right) \text{ or } \left(\sum_j C_j Z_j \le D\right) then \left(\sum_k E_k W_k \le F\right)$$

where X_i, Z_j and W_k are continuous variables; and A_i, C_j, E_k, B, D and F are given parameter vectors. [10]

b) Three power stations, A, B and C are committed to meeting the electricity demand of *1200 MW*. The fuel cost function, F_j, of each power station, j, can be described by a linear function of output of power station, X, P_j *(MW)*, as:

$$F_j(P_j) = 500 + 10P_j$$

Each power station is characterised by a number of allowed operating power ranges (due to physical operation limitations) as shown in the table below:

Power Station	**Power Range 1 (MW)**	**Power Range 2 (MW)**
A	*[100-200]*	*[300-500]*
B	*[150-250]*	*[400-600]*
C	*[150-300]*	*[350-450]*

Formulate the above problem as a mixed integer linear programming (MILP) model without solving it so as to select the optimal operation of power stations to minimise total fuel cost. Assume that there is no power loss. [15]

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5.

a) Reformulate the following bilinear terms, $X_i.Y_j$, of continuous (X_i) and binary (Y_j) variables to linear terms by introducing new variables and additional constraints. [5]

b)

i) For Flexibility Analysis, what are the advantages of the Active Set Method over the Vertex Enumeration Method? [4]

ii) Consider the following model of a system:

$$
\begin{aligned}
h_1 &= x_1 + 3x_2 - \theta_1 + \theta_2 + z = 0 \\
h_2 &= x_1 + 5x_2 + \theta_1 - 3\theta_2 - z = 0 \\
g_1 &= x_1 - x_2 + 5\theta_1 - 2\theta_2 - 2z \leq 0 \\
g_2 &= 3x_1 + 7x_2 - 2\theta_1 + 9\theta_2 + 3z \leq 0
\end{aligned}
$$

where x_1 and x_2 are the state variables, θ_1 and θ_2 are the uncertain parameters and z is the control variable. Eliminate the state variables and obtain the inequalities in terms of the uncertain parameters and control variables. [6]

iii) Formulate the Flexibility Test problem by using the Active Set Method strategy for the inequalities obtained in ii) above. Also, consider the following bounds on the uncertain parameters in the formulation:

$$
\begin{aligned}
0 \leq \theta_1 \leq 10 \\
0 \leq \theta_2 \leq 10
\end{aligned}
$$

[10]

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