UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sc.

M6: Advanced Process Engineering

COURSE CODE	:	**CENG00M6**
DATE	:	**24-MAY-05**
TIME	:	**10.00**
TIME ALLOWED	:	**3 Hours**

05-N0013-3-40

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***Answer FOUR questions. Each question carries a total of 25 marks, distributed as shown* [ ]. Only the *FIRST FOUR ANSWERS* will be marked.**

1.

a) Explain the difference between the steepest descent step and a Newton step in gradient based methods for unconstrained optimisation. What are the strengths and weaknesses of each type of step? [5]

b) Explain what is meant by the Secant relation in Quasi-Newton methods. [4]

c) Describe the three line search methods: exact line search, approximate line search, and the trust region method. [6]

d) Using a Newton based algorithm generate the first iteration without line search for the following problem starting from $x^0 = [1\ 1]^T$

$$f(x) = x_1^2 + x_1^2 x_2 - x_2^3$$

Is this first step a descent step? [10]

2.

a) What is meant by the Lagrangian of a constrained optimisation problem? [2]

b) Form the Lagrangian for the following problem and solve it analytically:

Minimise $(x_1^2 + x_2^2)$
Subject to $3x_1^2 + 2x_1x_2 + 3x_2^2 = 5$

Show your results graphically

[10]

c) What is meant by a feasible point? Give a feasible point for the problem in part b). [3]

d) At the heart of the SQP method is the need to solve a Quadratic Programming problem. What is a quadratic programming problem and how is it solved? [4]

e) What is meant by a projection in the active set method for solving linearly constrained optimisation problems? How is the projection of the Hessian matrix obtained? [6]

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3.

a) Consider a company which examines six candidate projects for execution over a period of three years. The expected expenditure over each year and cumulative profit for each project (in £k) are shown in the following table:

		Expenditure		
Project	**Year 1**	**Year 2**	**Year 3**	**Profit**
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
6	9	7	8	25

Assume that each project will be executed over the 3-year period if it is selected. Also, up to £25k can be invested over each individual year on the selected projects. The objective is to choose up to three projects so as to maximise the total profit of the company.
Formulate and solve the above zero-one integer programming problem.

[10]

b) Consider the following set of mathematical constraints:

$$0 \le B_{jt} \le V_j \cdot N_{jt} \quad \forall j,t$$

$$V_j = \sum_k (\bar{V}_{jk} \cdot E_{jk}) \quad \forall j$$

$$\sum_k E_{jk} = 1 \quad \forall j$$

where V_j is the capacity of unit j; N_{jt}, B_{jt} are the number of batches and the amount of material being processed, respectively, in unit j over time period t. These variables (V_j, N_{jt} and B_{jt}) are continuous ones. The E_{jk} binary (0-1) variables are then introduced which have the value of 1 if type(size) k is selected for unit j; 0 otherwise. The values of unit types(sizes), $\bar{V}_{jk}$, are assumed to be known.
The above constraints involve non-linear terms. Reformulate the above set of constraints to a mixed integer linear set by introducing new variables and additional constraints.

[15]

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4.

a) Let X_t be a binary variable denoting whether or not production takes place over time period t. If the product starts being manufactured at period t (*i.e.* $X_t=1$ and $X_{t-1}=0$) then production should be forced to continue for the next $m-1$ periods, thus keeping an uninterrupted production of *at least m* time periods. Formulate the above statement mathematically as a mixed integer linear constraint. [7]

b) Consider the following plant location-transportation problem. Given are $1..m$ clients (markets) with minimum product demand for each market j, D_j. There is one product being manufactured at production plants which can be built at a number of *possible* locations ($1..n$). If we decide to build a production plant at location i, then the maximum production capacity has to be A_i, and the associated fixed cost, C_i, should be included in the objective function. Also, the unit shipping cost (cost per shipped amount) from location i to market j is H_{ij}.

The problem is then to determine where to build the production plants and how much material should be shipped from selected locations to each market so as to minimise the total cost while satisfying minimum demand. Formulate the above problem as a mixed integer linear programming model. [18]

5.

a) A blending company can purchase up to five ingredients: *A, B, C, D* and *E*. Formulate the following cases as mixed integer linear programming constraints:

i) Select up to four out of five ingredients.
ii) Select at least three ingredients.
iii) If ingredient *E* is selected then *A* or *B* should be chosen as well.
iv) If only *B* or *C* is chosen, then ingredient *A* should not be selected

[5]

b) In the heat exchanger network shown in figure 1, the inlet temperatures, T_1 and T_3, of the hot (H1) and cold (C1) process streams are regarded as uncertain parameters. Given the nominal values of the temperatures shown and expected deviations of ±10 K in each of these streams:

i) Write a heat balance for the network. [2]
ii) Write temperature specifications for $\Delta T_{min} = 10K$. [2]
iii) Formulate the inequality constraints in terms of T_1, T_3 and the cooling load Q_c by eliminating the temperatures T_2 and T_4. [4]
iv) Apply the vertex enumeration scheme to determine the minimum value of Q_c for which the network is feasible over the entire uncertainty region. [12]

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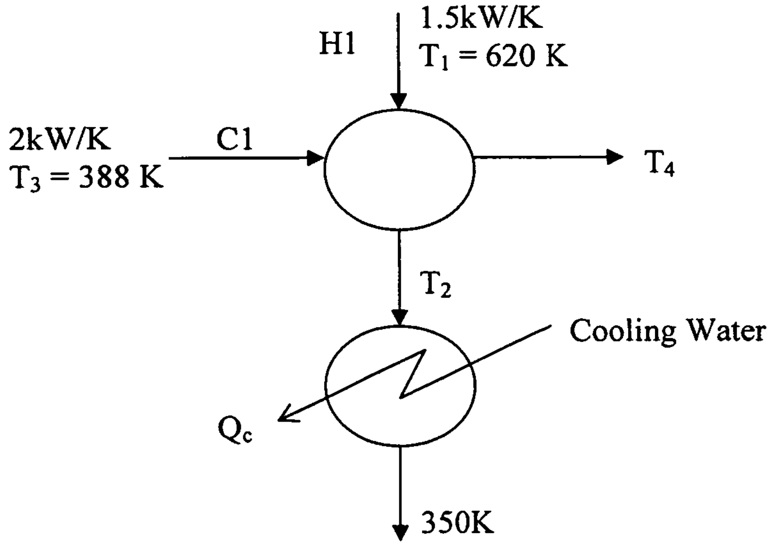

Figure 1

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