UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sc.

M6: Advanced Process Engineering

COURSE CODE	:	**CENG00M6**
DATE	:	**11-MAY-04**
TIME	:	**10.00**
TIME ALLOWED	:	**3 Hours**

04-N0011-3-40

TURN OVER

***Answer FOUR questions. Each question carries a total of 25 marks, distributed as shown* [ ]. Only the *FIRST FOUR ANSWERS* will be marked.**

1.

a) The basic iterative algorithm for gradient based algorithms for solving unconstrained optimisation problems is given below:

1. Choose initial point
2. Determine step direction
3. Determine step length
4. Update information
5. If convergence attained stop

if not return to step 2

Discuss each of these steps in turn giving mathematical formulations where possible that can be used in implementation. [8]

b) Using a Newton based algorithm generate the first iteration without line search for the following problem starting from $\boldsymbol{x}^0 = [-1 \ \ 1]^T$

$$f(x) = x_1^3 + x_1x_2 - x_2^2x_1^2$$

[10]

c) What condition is required to ensure that steps generated by Newton type methods are descent steps? [2]

d) What properties of the Hessian matrix does the BFGS (Broyden-Fletcher-Goldfarb-Shanno) update retain from iteration to iteration? Explain why each property is important. [5]

2.

a) The basic steps of the Successive Quadratic Programming algorithm for solving constrained optimisation problems are as follows

1. Choose an initial feasible point
2. Solve the Quadratic programming problem for the step direction
3. Line search
4. Solve for the Lagrange multipliers
5. Evaluate the gradients. If the point is a Kuhn Tucker point then stop.
6. Update the Hessian approximation using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) update
7. Set $k = k+1$ and return to step 2.

i) What are the Kuhn Tucker conditions?
ii) What is meant by a Quadratic Programming Problem?
iii) Explain the significance of the Lagrange multipliers?
iv) Give an example of a method that may be used for the line search.

[10]

CONTINUED

b) The cost of constructing a distillation column can be written as

$$C = C_p AN + C_s HAN + C_f + C_L$$

where

C = total cost	£
C_p = cost per square metre of plate area	500 £ m^{-2}
N = number of plates	
C_s = cost of shell	200 £ m^{-3}
H = distance between plates	0.5 m
A = column cross sectional area	m^2
C_L = cost of reflux pump	£
C_f = other fixed costs	£30000
L = reflux rate	kg m^{-3}
D = distillate rate	kg m^{-3}

There are three further equations relating the variables

$$A = 0.01(L + D)$$

$$L/D = N/(N - 5)$$

$$C_L = 5000 + 0.7L$$

Suggest two ways that the problem can be solved analytically (it is not necessary to solve the problem). [10]

c) What is meant by a non-convex constraint? [5]

3.

a) The following is a mixed integer non-linear programming model involving bilinear products of continuous (X_{ij}) and binary (Y_{ij}) variables:

$$\min z = \sum_i \sum_j X_{ij}$$

subject to

$$\sum_j X_{ij} Y_{ij} \le A_i \qquad \forall i$$

$$Y_{ij} \in \{0,1\}$$

$$X_{ij} \ge 0$$

where A_i are given parameters.

CONTINUED

Reformulate the model to a linear one by introducing new variables and additional constraints. [10]

b) Three power stations, A, B and C are committed to meeting the electricity demand over two different time periods during the day: 2500 MW during first period and 3500 MW during second period. A power station started in the first time period can be used in the second period without incurring any additional start-up cost. All power stations are turned off at the end the day. All related costs are shown in the table below:

Power Station	**Max Capacity (MW)**	**Start-up Cost**	**Fixed Cost per Period**	**Cost per Period per MW used**
A	2000	6000	500	6
B	1500	4000	600	4
C	3000	2000	700	8

Formulate the above problem as a mixed integer linear programming (MILP) model without solving it so as to select which power stations should be working during each time period to minimise total cost. [15]

4.

a) Formulate the following logical implication by a mixed integer linear set of constraints by using 0-1 variables:

$$\text{If } \sum_i A_i X_i \le B \text{ then } \sum_j C_j Z_j \le D$$

where X and Z are continuous variables; and A_i, C_j, B and D are given parameters vectors. [10]

b) Consider the following two non-linear functions of positive $X \ge 0$:

$$f(X_1, X_2, X_3) = X_1 X_2 + X_1^{0.6} + \frac{X_1}{X_3}$$

$$g(X_1, X_2, X_3, X_4) = X_1 X_2 - X_3 X_4$$

i) Are these functions convex? Explain. [3]

ii) Can the above functions be convexified by using suitable variable transformations? [4]

CONTINUED

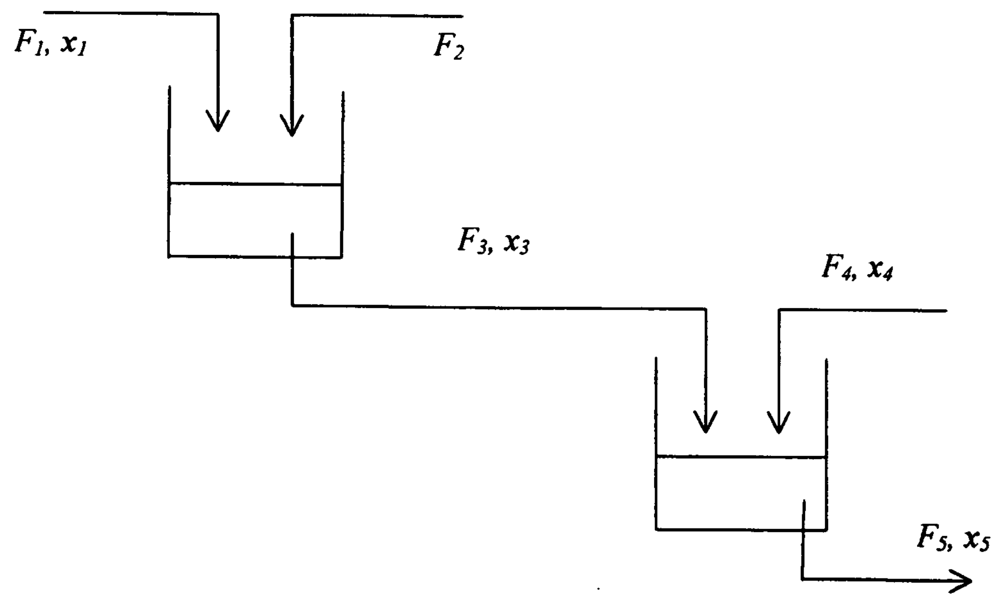
c) Let Y_{ij} denote the existence of a heat exchanger between streams i and j. Formulate the following statement as a set of constraints that are linear in Y_{ij}:

"A heat exchanger between streams i and j with area A_{ij} has a fixed cost F and a variable cost V, where $V = C \times A_{ij}^{0.6}$ and C is a constant. The area of the exchanger should be larger than L and smaller than U" [8]

5.

a) Many optimisation problems involving integer variables are solved iteratively (for example, Outer-Approximation). Usually, one common step of these solution algorithms is the introduction of an extra constraint (integer cut) at each iteration in the master problem (0-1 programming problem) in order to make infeasible the choice of binary vectors obtained from previous iterations. Develop such a constraint. [5]

b) Consider the process shown in the diagram below. A methanol solution (methanol mole fraction: x_1) with flowrate F_1 is mixed in a tank with a pure methanol stream of flowrate F_2. The tank outlet has a flowrate F_3 and methanol mole fraction x_3. It is mixed in a second tank with a 45% methanol solution (on a mole basis) of flowrate F_4. The product of the second tank has flowrate F_5 and methanol mole fraction x_5.



Process data:

$F_1 = 10$ kmol/hr; $x_4 = 0.45$

Uncertain parameters: $0.2 \le x_1 \le 0.4$ (nominal value: 0.3)
$1 \le F_2 \le 3$ (nominal value: 2 kmol/hr)

Control variable: $F_4 \ge 0$

CONTINUED

Performance targets: $F_5 \geq 13$ kmol hr^{-1}

$0.4 \leq x_5 \leq 0.5$

i) Derive a steady state model for this process which relates all inputs, uncertain parameters, and outputs. Formulate the model so that it contains inequality constraints only. [8]

ii) Is the process feasible for the entire uncertainty region?

1) if $F_4 \leq 30$ kmol hr^{-1}
2) if $F_4 \leq 24$ kmol hr^{-1} [12]

END OF PAPER