



Answer **THREE** questions including Question 1

The marks for each question are distributed as shown [ ]

1. a) Explain the differences between a sequential-modular package and an equation-oriented package. Give an advantage of each. [5]

b) Runge-Kutta Methods are useful for solving ordinary differential equations. List the formulae involved in the Runge-Kutta Second Order and Fourth Order Methods to solve ODE problems. Explain the importance of the selection of the step size  $h$  for Runge-Kutta Methods. [5]

c) A biotech company manufactures and sells 3 proteins used in drinks with added health benefits: 'memory-booster protein', 'improved-concentration protein' and 'stress-relief protein'. The fermentation media for each protein are prepared using three common ingredients: A, B and C. In a given period there are 35 kg of A, 50 kg of B and 45 kg of C available. The amount of these ingredients required per batch, as well as the profit per batch, for each protein are shown in the table below:

	Memory-booster protein	Improved-concentration protein	Stress-relief protein
A (kg)	2	2	1
B (kg)	2	3	3
C (kg)	2	4	1
Profit	£400	£500	£400

The company wishes to determine how many batches of each protein it should make so as to maximise its profit and satisfy the constraints.

i) Let  $x_1$  be the number of batches of 'memory-booster protein',  $x_2$  be the number of batches of 'improved-concentration protein' and  $x_3$  be the number of batches of 'stress-relief protein'. Formulate the objective function and possible constraints as a linear programming problem. [Do NOT solve the problem.] [3]

ii) Construct the initial Simplex tableau, identifying the basic variables. [1]

iii) What would the new non-basic and basic variables be in the next iteration of the Simplex method? [2]

iv) The final tableau is given below.

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	P	b
$x_1$	1	0	0	1.125	-0.25	-0.375	0	10
$x_2$	0	0	1	-0.25	0.5	-0.25	0	5
$x_3$	0	1	0	-0.5	0	0.5	0	5
P	0	0	0	100	100	0	1	8500

How many batches of each protein should be produced? What is the optimal profit?

[2]

**CONTINUED**

v) The company is considering increasing its stock supply of one of the ingredients. Would you recommend they increase the amount of A, B, or C? Explain your answer. [2]

d) Explain the function of 'SCENARIO' in EXCEL. List the benefits of using this function. Explain the basic requirements needed in order to utilise this function in a spreadsheet. [5]

2. a) Explain the difference between the Implicit Method and Explicit Method for solving ODE problems. Give an example and an advantage of each method. [5]

b) The following MATLAB program is used to plot the results of two different functions  $y_1$  and  $y_2$  where  $y_1 = e^{-x} \sin x$  and  $y_2 = 5t$  respectively. There are five errors in the program. Find these errors and give suggestions to correct them. [5]

\$ Program to Question 2. b)

```
subplot(211)
x=0:-0.1:3
y1=exp(-x)*sin(x)
plot(x,y1,'o')
title('y1=exp(-x) sinx')
axis([0 3 0 0.5])
```

```
subplot(222)
plot(t,5*t,'*')
title('y2=5t')
```

c) Given that

$$\frac{dy}{dt} = 2y + t^2 + 1$$
$$y(0) = 4$$

Select a numerical method to solve this ODE system and write a MATLAB program to solve the problem and to display the results for  $0 \leq t \leq 5$ . [10]

**TURN OVER**

3. a) The selection of an initial guess point is important when using the Newton-Raphson Method to solve a non-linear equation. Suggest three methods to obtain a suitable initial guess point. Describe how you would use these methods to approximate an initial guess point for the following non-linear equation:

$$f(x) = x^3 + 10x + 10 = 0$$

[10]

- b) Given the following system of non-linear equations:

$$\begin{aligned} x_1x_2 - x_2 + 10 &= 0 \\ x_2^2 - 2x_1 + 23 &= 0 \end{aligned}$$

- i) Determine the Jacobian matrix for this system of equations. [3]
- ii) Obtain the new iterate values of  $x_1$  and  $x_2$  using the Newton-Raphson Method for this problem, starting from the initial point

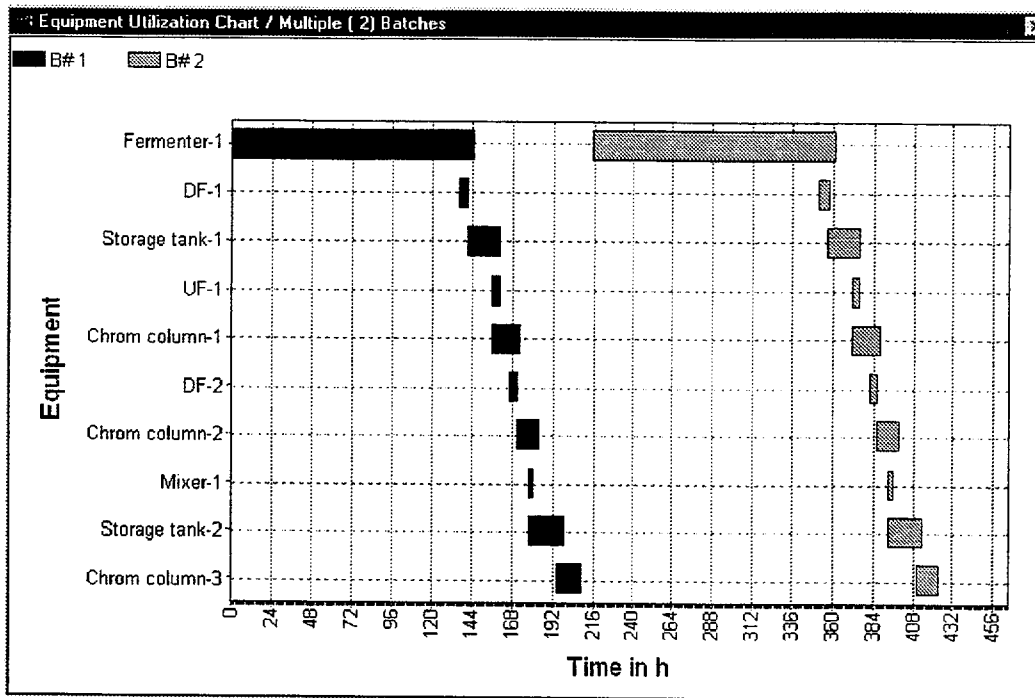
$$x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

[7]

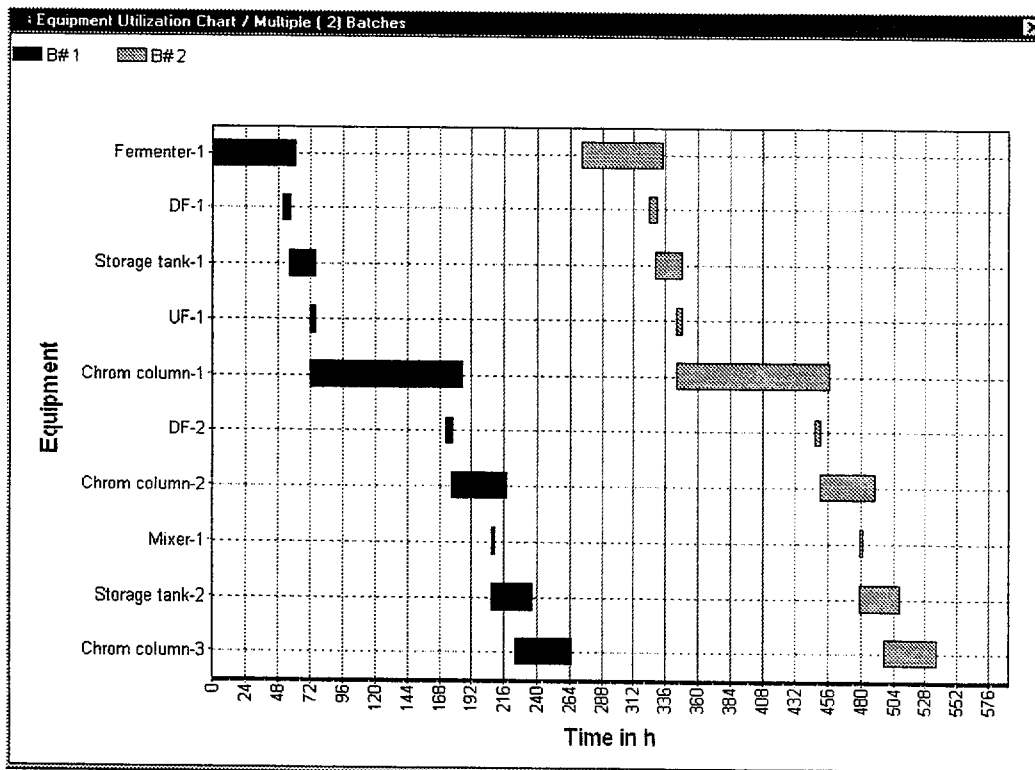
4. a) Explain the difference between a design problem and a rating/simulation problem. Give examples of the key inputs you would specify in each case for a microfiltration unit. [6]
- b) Give three examples of activities that must occur during a manufacturing campaign in addition to the process steps that handle the product. [3]
- c) A process for manufacturing antibody fragments in *E.coli* was modelled using SuperPro Designer. The following scheduling information for a single campaign was computed:
- Actual batch time = 254 hours  
Minimum effective batch time = 64 hours.
- Given an annual operating time of 7900 hours, determine whether this plant can complete 100 batches a year. [3]
- d) A biopharmaceutical company manufactures two products in two plants, Plant 1 and Plant 2. Fermentation operators and purification operators are employed on an hourly basis. SuperPro Designer was used to set up the flowsheet of the manufacturing process in each plant. The Equipment Utilisation charts generated by SuperPro Designer for each plant are shown on the next page.
- i) Identify the equipment bottleneck in the process for Plant 1 and Plant 2. [2]
- ii) The manufacturing team wishes to increase the throughput in each plant. Provide a suggestion of what it can do in each case. Indicate an advantage and a possible disadvantage of your suggestion in each case. [6]

**CONTINUED**

Plant 1:



Plant 2:



END OF PAPER