Answer THREE questions.

1.
Consider the curve defined by the following rational polynomials:

$$x(t) = \frac{1 + 3t + 6t^2 + 2t^3}{1 + 6t - 3t^2}, \quad y(t) = \frac{3 + 12t^2 - 3t^3}{1 + 6t - 3t^2}, \; t \in [0,1]$$

(a) Find the polar forms for each of the three polynomials given.

[5 Marks]

(b) Find the control points and weights for the representation of this curve in the cubic rational Bezier form.

[5 Marks]

(c) Consider the curve defined by the numerators only of x(t) and y(t). Suppose that this curve is to be represented in cubic B-Spline form, on the knot sequence 1, 2, 3, 4, 5, 6. Describe how to find the B-Spline control points, and find the first one.

[7 Marks]

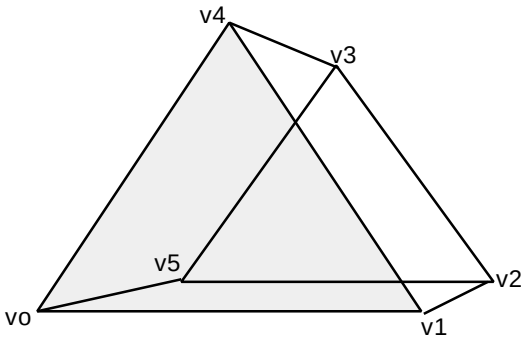
(d) Describe in detail how to insert a new knot 3.5 into the sequence.

[8 Marks]

2.
Consider the shape below to be represented using the Winged Edge data structure.

(a) For edges 0 and 4 indicate the Next Vertex, Previous Vertex, Next Face and Previous Face for the Winged Edge data structure.

[7 Marks]



$v_0\ v_1\ v_4 = Face_0$
$v_5\ v_3\ v_2 = Face_1$
$v_1\ v_2\ v_3\ v_4 = Face_2$
$v_0\ v_4\ v_3\ v_5 = Face_3$
$v_0\ v_5\ v_2\ v_1 = Face_4$

$edge_0 = v_0\ v_1$
$edge_1 = v_1\ v_2$
$edge_2 = v_2\ v_5$
$edge_3 = v_5\ v_0$
$edge_4 = v_1\ v_4$
$edge_5 = v_2\ v_3$
$edge_6 = v_5\ v_3$
$edge_7 = v_0\ v_4$

(b) Explain how the winged edge data structure would be completed - that is how the wings would be found. Give an example using edges 0 and 4.

[9 Marks]

(c) Describe how to build a winged edge data structure given a vertex-face data structure.

[9 Marks]

3.

(a) Define the usual parameters that control a virtual camera (eg, the View Reference Point - VRP - is one such parameter).

[7 Marks]

(b) Given the parameters in (a) describe in detail the derivation of the matrix that transforms an object's coordinates from World Coordinates to a coordinate system about the camera (the Viewing Coordinate System) for a perspective projection.

[10 Marks]

(c) Suppose there are three distinct points in 3D space p_0, p_1 and p_2. A virtual camera is moving along the paths from p_i to p_{i+1} (i=0,1,2) with $p_n \equiv p_{n \bmod 3}$. When the camera is on the path p_i to p_{i+1} it is "looking" along the corresponding vector. When the camera reaches the end of a particular path, it stops, and gradually rotates through the angle defined by the successive vectors, until it is facing in the direction given by the next vector. The up-direction of the camera is chosen so that it is perpendicular to the plane defined by the three points. Describe an algorithm for moving the camera continuously through this path, carefully describing any functions that you assume.

[8 Marks]

4.

(a) Describe how images are computed in ray-tracing. Give an outline of the algorithm for tracing a ray.

[8 Marks]

(b) Explain the adaptive depth control for terminating the recursion of rays.

[6 Marks]

(c) Describe methods for accelerating the ray-scene intersection tests.

[6 Marks]

(d) What are the main differences between ray-tracing and path tracing.

[5 Marks]

5.

(a) Outline the basic radiosity algorithm. Give the radiosity equation and explain its terms.

[8 Marks]

(b) How does progressive refinement (PR) radiosity differ from the full matrix radiosity? What is the role of the ambient term in PR? How is it calculated and how is it used?

[12 Marks]

(c) Explain how adaptive subdivision can be used to improve the radiosity solution.

[5 Marks]