## Answer THREE questions

Calculators are permitted
1.

The management of Standstill Airport are considering modernising the main terminal. At the moment aircraft are parked on the apron in front of the terminal building and passengers disembark onto the tarmac and walk to the building. The plan is to replace this system with a modern complex of gates and umbilical gangways. Two types of gangways will be needed, one for large aircraft and another for smaller planes.

The management needs a model that will help them to evaluate the number of gangways needed of each type. The evaluation should be based on existing assessments of future traffic patterns for both passengers and aircraft. A major problem is that the new system will allow space for only five planes waiting for takeoff and two waiting for a gate. A third waiting plane would block the runway and prohibit further takeoffs and landings.
a) Outline the main characteristics of discrete event simulation.
b) Give a detailed description of a model that will allow investigators to evaluate the proposed new layout using discrete event simulation.
c) Outline the main characteristics of Monte Carlo simulations.
d) How would the model change if Monte Carlo evaluation were to be used rather than discrete event simulation?
e) Discuss the merits of the two models.
2.


The Century Dome exhibition hall operates the entrance system shown above with seven entrance points. The time needed to admit a visitor is exponentially distributed with an average of 5 seconds. After registration, the visitors join a single queue before passing through a security check. The time to perform a security check is assumed to be exponentially distributed and will on average take 72 seconds. All queues have infinite capacity.
During the afternoon, the exhibition is visited by 3,000 people per hour. We assume that the arrival rate is Poisson distributed and the system reaches a steady state.
a) How many people will on average be queueing for registration?
[8 marks]
b) What will the average queueing time be if 80 security check points are provided by the organisers? (Table listing values of $\pi_{0}$ for 80 servers is enclosed)
c) Queueing for the most popular exhibit in the dome is restricted to a maximum of 50 visitors. What is the probability of finding the queue full if a visitor can enter every 1.2 second and no visitor is allowed to return to the queue?
3.

The Empire has three parties, the Imperialists, the Rebels and the Yellow party. A poll has predicted the following voting patterns for the electorate:

| Vote in <br> Last election | Prob. of voting <br> Imperialist next | Prob. of voting <br> Rebel next | Prob. of voting <br> Yellow next |
| :--- | :---: | :---: | :---: |
| Imperialist | 0.5 | 0.1 | 0.4 |
| Rebel | 0.1 | 0.7 | 0.2 |
| Yellow | 0.3 | 0.3 | 0.4 |

In the last election $56 \%$ of the electorate voted for the Imperialists, $38 \%$ voted for the Rebels and $6 \%$ voted for the Yellow party
a) Assuming the poll is a correct representation of voting intentions, would you expect the Imperialists to retain their absolute majority after two further elections?
[11 marks]
b) If the voting intentions remain fixed, what will the Rebel's steady state share of the votes be?
[8 marks]
c) If the voting intentions remain fixed and a person voted for the Rebels in the last election. How many elections will take place on average before he votes for the Rebels again?
d) If the transition probability matrix changed to

| Vote in <br> Last election | Prob. of voting <br> Imperialist next | Prob. of voting <br> Rebel next | Prob. of voting <br> Yellow next |
| :--- | :---: | :---: | :---: |
| Emperialist | 0.8 | 0.1 | 0.1 |
| Rebel | 0.0 | 0.8 | 0.2 |
| Yellow | 0.0 | 0.5 | 0.5 |

by how much would the steady state probability for a Rebel vote change?
4.
a) Explain the basic features of a dynamic programming problem, and outline typical areas where the technique can be applied.
[12 marks]
b)

| From | To | Cost |
| :--- | :--- | ---: |
| Edinburgh | Newcastle | 13.90 |
| Edinburgh | Carlisle | 10.30 |
| Carlisle | Leeds | 5.00 |
| Carlisle | Preston | 5.80 |
| Carlisle | York | 7.90 |
| Newcastle | Leeds | 9.40 |
| Newcastle | Preston | 7.90 |
| Newcastle | York | 3.70 |
| York | Crewe | 4.30 |
| York | Hull | 3.50 |
| York | Sheffield | 3.80 |
| Leeds | Crewe | 2.90 |
| Leeds | Hull | 4.00 |
| Leeds | Sheffield | 2.50 |
| Preston | Crewe | 2.20 |


| From | To | Cost |
| :--- | :--- | ---: |
| Preston | Hull | 4.00 |
| Preston | Sheffield | 3.20 |
| Crewe | Peterborough | 5.40 |
| Crewe | Derby | 3.30 |
| Crewe | Birmingham | 2.50 |
| Hull | Peterborough | 2.20 |
| Hull | Derby | 2.30 |
| Hull | Birmingham | 4.70 |
| Sheffield | Peterborough | 3.30 |
| Sheffield | Derby | 2.00 |
| Sheffield | Birmingham | 2.30 |
| Peterborough | London | 7.70 |
| Derby | London | 6.90 |
| Birmingham | London | 5.80 |
|  |  |  |

The above chart shows the bus fares for travel between a number of British cities. Use dynamic programming to determine the cheapest route from Edinburgh to London.
[12 marks]
c) Discuss the computational efficiency of dynamic programming with special reference to the above example.
5.


The finite element shown above has a thickness of $\tau=2 \mathrm{~mm}$. Its coefficient of thermal expansion is $\alpha=2 * 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ and the change in temperature is $\Delta \mathrm{T}=50^{\circ} \mathrm{C}$. and the pressure acting on the surface $(i, j)$ is $P=10 \mathrm{~N} / \mathrm{mm}^{2}$. The element is of the plane stress type with

$$
[D]=\frac{E}{1+v^{2}}\left[\begin{array}{ccc}
1 & v & 0 \\
v & 1 & 0 \\
0 & 0 & \frac{1-v}{2}
\end{array}\right]
$$

where Young's modulus is $\mathrm{E}=200 \mathrm{GPa}$ and Poisson's ratio is $v=0.3$.
a) Determine the shape functions associated with the element.
b) Determine the matrix ([B]) that can be used to equate the strain vector $\{\varepsilon\}$ and the nodal displacement $\{\mathrm{U}\}$ as $\{\varepsilon\}=[\mathrm{B}]\{\mathrm{U}\}$.
c) The displacements are found to be:

| Node | $u(\mathrm{~mm})$ | $\mathrm{v}(\mathrm{mm})$ |
| :---: | ---: | ---: |
| i | 2.0 | 0.0 |
| j | 0.5 | 2.0 |
| k | 1.0 | -0.5 |

Determine the stresses predicted in the element.
d) Determine the total strain in the z-direction

