Answer THREE questions

Calculators are permitted

1.

The maintenance division of a van rental company consists of three sections:

1. Valet service,
2. Mechanical repair workshop
3. Body repair workshop.

Cars returned by customers are evaluated by a works office where it is decided what action to take. Each van waits on average 20 minutes in the works office while the decision is made. Of the vans passing the works office

25% are released to customers. On average, a customer will use the van for 7 hours before returning it.

50% are directed to the valet service. The average treatment takes 40 minutes. After completion, the vans are returned to the works office.

22% have mechanical faults and are directed to the workshop. The repairs take on average 2 hours. After completion, the vans are returned to the works office.

3% have damaged bodywork and are directed to the body repair workshop. Repairs take on average 16 hours. After completion, the vans are forwarded directly to the valet service.

a) Draw and label a diagram of the maintenance division.

[5 marks]

b) For each van, what are the respective visit ratios for the works office and each of the three sections?

[7 marks]

c) What is the total service demand for each division?

[7 marks]

d) Where is the bottleneck?

[7 marks]

e) If the utilisation of the valet service is 80%, what is the average response time if there are 100 customers?

[7 marks]

2.

A mini-survey of 10 women's drinking habits gave the following results:

Age	Units per week	Weight
22	15	57.0
33	12	58.0
26	20	58.4
40	10	58.7
18	25	59.0
35	17	59.2
30	22	60.0
24	27	60.5
20	30	60.7

a) Explain the principle of least square estimation.

[6 marks]

b) Give a detailed explanation of how the least square principles can be used to determine correlation between the variables in the survey.

[13 marks]

c) Determine the relationship between a woman's alcohol intake (units per week) and her weight using the data from the mini-survey.

[14 marks]

3.

a) Describe the main characteristics of a stochastic state machine.

[5 marks]

b) Give a detailed description of how a stochastic state machine can be evaluated using discrete event simulation techniques.

[5 marks]

The management of Standstill Airport are considering modernising the main terminal. At the moment aircraft are parked on the apron in front of the terminal building and passengers disembark onto the tarmac and walk to the building. The plan is to replace this system with a modern complex of gates and umbilical gangways. Two types of gangways will be needed, one for large aircraft and another for smaller planes.

The management needs a model which will help them, based on current traffic records and expansion projections, to evaluate the number of gangways needed of each type. A major problem is that the new system will allow space for only five planes waiting for takeoff and two waiting for a gate. A third waiting plane would block the runway and prohibit further takeoffs and landings.

c) Construct a model that will use discrete event simulation to evaluate the suitability of the proposed scheme.

[15 marks]

d) Give a detailed account of how your model can be used to provide a realistic analysis of the working of the proposed airport changes.

[8 marks]

4.

A MORI poll has found the following voting patterns for the British electorate:

Vote in Last election	Prob. voting Conservative next	Prob. voting Labour next	Prob. voting Liberal next
Conservative	0.6	0.1	0.3
Labour	0.1	0.7	0.2
Liberal	0.4	0.1	0.5

In the last election 56% of the electorate voted Conservative, 38% voted Labour and 6% voted Liberal

a) What is Labour's steady state share of the votes?

[8 marks]

b) From the above data, would you expect the Conservatives to have a majority after two further elections?

[9 marks]

c) If a person voted Labour in the last election, how many elections will take place on average before he votes Labour again?

[9 marks]

If the transition probability matrix changed to

Vote in Last election	Prob. voting Conservative next	Prob. voting Labour next	Prob. voting Liberal next
Conservative	0.8	0.1	0.1
Labour	0.0	0.8	0.2
Liberal	0.0	0.5	0.5

d) By how much would the steady state probability for a Liberal vote change?
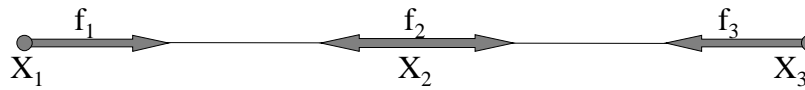
[7 marks]

5.

For the finite element representation of a one-dimensional pin-jointed bar, the load-displacement relationship is given by

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\alpha}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

where $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$ are the forces at the two ends of the bar, $\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$ the corresponding displacements, L the length of the bar and α a material constant.
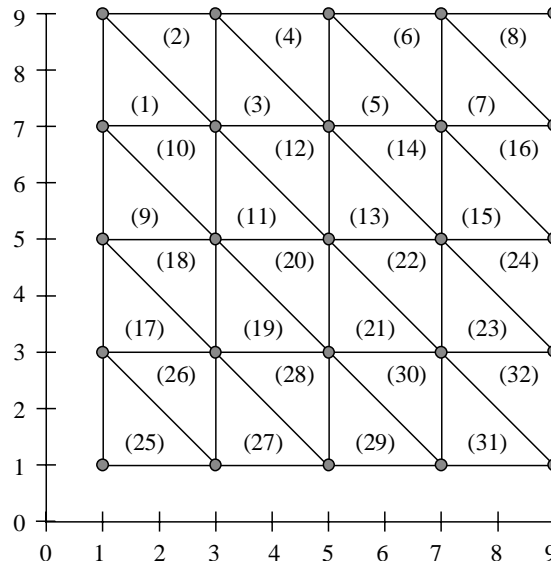

a) Derive the corresponding expression for a system of two bars where the length of one bar is $L_1 = 1$ cm, the length of the other $L_2 = 2$ cm and the material constants are $\alpha_1 = 3$ and $\alpha_2 = 8$ respectively.

[15 marks]

b) Explain what shape functions are, how they are used and how they relate to natural co-ordinates.

[7 marks]

c) Calculate the shape functions for elements (14) and (19) in the mesh shown above.

[9 marks]