Exam Questions for B21b, section B

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SECTION B

4. Recall that the *fibonnacci sequence* $1, 1, 2, 3, 5, 8, 13, \ldots$ is defined by $fib(1) = 1,\ fib(2) = 1$ and for $n > 2,\ fib(n) = fib(n-1) + fib(n-2)$.

   a. Consider this program:-

   f(1):= 1

   f(2):= 1

   f(n)

   $\{$

   s:= 1

   for ($i := 1$ to $n - 2$) $s := s + f(i)$

   return(s)

   $\}$

   What function does the program f define?

   [7 marks]

   b. By defining a suitable variant for f prove that this program terminates.

   [9 marks]

   c. Modify the program f above to obtain a program g that computes the more general fibonnacci sequence $fibo$ defined by $fibo(1) = a,\ fibo(2) = b$ (a and b are arbitrary numbers) and for $n > 2,\ fibo(n) = fibo(n-1) + fibo(n-2)$.

   [5 marks]

   d. Prove, by induction or otherwise, for all $n \geq 1$ that $g(n) = fibo(n)$.

   [12 marks]

[Total = 33 marks]

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5. a. Write pre- and post-conditions for a program that takes a list of non-negative integers and returns the list sorted in ascending order.

[4 marks]

Consider the following program, written in pseudo-code. Let L be a list of non-negative integers $L = [L(0), L(1), \ldots, L(m-1)]$, for some $m \geq 0$ and some non-negative integers $L(0), \ldots, L(m-1)$.

```
Slow_sort(L)
{ while (Sorted(L) = false)
    { pick i < j < m with L(i) > L(j)
        Swap(L, i, j)
    }
}
```

"Sorted" is supposed to be a boolean function that returns 'true' if its argument is a list sorted in ascending order and 'false' otherwise. "Swap(L, i, j)" is supposed to be a function that swaps $L(i)$ and $L(j)$ in the list L and leaves all other elements alone.

b. Write clear pre and post-conditions for "Sorted" and "Swap".

[4 marks]

c. Write an implementation of the functions "Sorted" and "Swap" in pseudo-code. (But you do not have to prove that they are correct).

[5 marks]

d. Assuming that "Sorted" and "Swap" are implemented correctly, prove that "Slow_sort" is weakly correct (i.e. if it terminates then it will meet its specifications).

[8 marks]

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e. Prove that the function "Slow_sort" (above) terminates. You may assume that "Sorted" and "Swap" terminate. You may find it helpful to consider the function $V(L) = L(0) + N \times L(1) + \ldots + N^i \times L(i) + \ldots + N^{m-1}L(m-1)$, where $|L| = k$ and N is some upper bound for the elements of the list, i.e. $0 \leq L(i) < N$ for $i = 0, 1, \ldots, m-1$.

[12 marks]

[Total = 33 marks]

6. Let S be a procedure and let wp_S denote the 'weakest precondition predicate transformer of S'.

   a. Give a precise definition of wp_S.

   [6 marks]

   b. Let P and Q be any postconditions. Find an equivalent formulation for $wp_S(P \wedge Q)$ in terms of $wp_S(P)$ and $wp_S(Q)$ only.

   [6 marks]

   c. Let S and T be any procedures and let Q be any postcondition. Let $S;T$ denote the procedure that consists of doing S first and then doing T. Find an equivalent formulation for $wp_{S;T}$ using the predicate transformers wp_S and wp_T only.

   [6 marks]

   d. Let i, j, k be integer valued variables and let S be the procedure:-

   $k := i$

   $j := k + 1$

   $i := 3 - j$

   Calculate

   1. $wp_S(i < 5)$
   2. $wp_S(0 < i < 3)$
   3. $wp_S(j = 1)$
   4. $wp_S(i = k)$
   5. $wp_S(i > j)$

   [15 marks]

[Total = 33 marks]

END OF PAPER