1. a. Let the binary predicate $arc(x, y)$ denote a directed arc from x to y, and let the uniary predicate $node(x)$ denote a node x. Using these predicates, give a formula of first-order logic for each of (i) to (iv) in order to define each of them as a constraint on a graph. Restrict the variables in the formulae to only being instantiated with nodes.

    i. A path.

    ii. A connected graph.

    iii. A directed acyclic graph.

    iv. A tree.

    b. Give the pseudocode for an algorithm that determines whether a graph (N, A) is a tree, where N is a set of nodes and A is a set of arcs. No other information about the graph can be assumed.

    c. Consider depth-first search.

    i. Give a recursive definition.

    ii. Give a defintion based on a stack.

[Total 33 marks]

2.   a. Compare and contrast the following approaches to designing algorithms:

   i. Greedy algorthms.

   ii. Divide and conquer.

   iii. Dynamic programming.

[15 marks]

   b. Consider Dijkstra's algorithm.

   i. What are the conditions on the all input to the algorithm?

   ii. What is the output from the algorithm?

   iii. Give an example of input to the algorithm that has five nodes and ten arcs, and give the output from the algorithm?

   iv. Explain in what way the algorithm is a greedy algorithm.

[10 marks]

   c. Explain why there is no greedy algorithm that is guaranteed to give an optimal solution for the travelling salesperson problem.

[8 marks]

[Total 33 marks]

3. a. i. Give an efficient format for representing binary search trees.

   ii. Give the definition for the binary search tree property.

   b. Give the psuedocode for an algorithm for inserting a new node into a binary search tree.

   c. Give the binary search tree that is formed from the following sequence of keys, assuming that the keys are inserted from left to right:

   $$15, 6, 18, 3, 2, 4, 7, 13, 9, 17, 20$$

   d. Explain how a key is deleted from a binary search tree.

   [Total 33 marks]

4. a. i. What is the aim of Huffman coding?

   ii. What is a trie?

   b. For the following piece of text, give a code trie formed using Huffman coding.

   GOOD APPLES AND ORANGES GROW IN THE OPEN
   IN INDONESIA INDIA AND SPAIN

   c. i. For a text with n different characters, what are the upper and lower bounds on the height of the trie formed using Huffman coding. Explain why.

   ii. Is the Huffman coding algorithm, a divide and conquer algorithm, a greedy algorithm, or a dynamic programming algorithm? Explain why.

   [Total 33 marks]

TURN OVER

5.　a.　Consider the following cryptomessage

ZNKFURJFJUMFZNKFLUUZYZKVYFULFZNKFDU TM

　　i.　What is the Caesar key?

　　ii.　What is the unencrypted message?

b.　How many different ways can a message be encoded using:

　　i.　The Caesar key approach?

　　ii.　Substitution tables

c.　How does a public-key cryptosystem work, and what is the advantage over other cryptosystems?

d.　Explain the RSA cryptosystem method including:

　　i.　How are the keys chosen?

　　ii.　How is a message encoded, and then decoded?

　　iii.　In what way is the method secure, and yet viable?

[Total 33 marks]

END OF PAPER