Computer Science Department

Dummy Cover Sheet

M.Sc. / Coll Dip

3c72 Image Processing, 2000-2001

2 hours 30 minutes

Bernard Buxton and Simon Arridge

*Answer **three** questions. Each question is worth 33 marks. The total time allowed is two and a half hours. The use of electronic calculators is permitted.*

Question 1

(a) Define the operation of *convolution* for two continuous one-dimensional functions $f(t), g(t), t \in (-\infty, \infty)$, and for two lists of sampled functions $\{f_j;\ j = 0....N-1\}$, $\{g_j;\ j = 0....M-1\}$, and state the convolution theorem in each case.

[5 marks]

(b) Show that if $h(t) = f(t) * g(t)$ then

$$\frac{dh(t)}{dt} = \frac{df(t)}{dt} * g(t) = f(t) * \frac{dg(t)}{dt}$$

where '*' represents convolution.

[3 marks]

(c) If a Gaussian convolution kernel of scale σ in two-dimensions is defined as:

$$g(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{x^2 + y^2}{2\sigma^2}\right],$$

find the form of the derivatives $g_x = \frac{\partial g(x, y)}{\partial x}$, $g_y = \frac{\partial g(x, y)}{\partial y}$, $g_{xx} = \frac{\partial^2 g(x, y)}{\partial x^2}$, $g_{xy} = \frac{\partial^2 g(x, y)}{\partial x \partial y}$ and $g_{yy} = \frac{\partial^2 g(x, y)}{\partial y^2}$, in both the spatial and Fourier domains. Sketch the forms of these functions.

[10 marks]

(d) If an image is considered as a function $f(x, y)$ show how the functions derived in part (b) can be convolved with $f(x, y)$ to calculate:

1. the *gradient* of $f(x, y)$ blurred by the Gaussian $g(x, y; \sigma)$,
2. the *Laplacian* of $f(x, y)$ blurred by the Gaussian $g(x, y; \sigma)$.

Your answer should explain how this can be done either using the spatial-domain functions or using the Fourier domain functions, and what the advantages of one method over the other might be.

[10 marks]

[Question 1 cont. over page]

[TURN OVER]

[Question 1 cont.]

(e) How can the zero-crossings of the Laplacian of an image be used as an edge detector ? What advantages are there in considering the Laplacian at more than one value of the blurring parameter σ ?

[5 marks]

[Total 33 marks]

Question 2

(a) Discuss how the general concept of *image filtering* can be considered as the multiplication of a vector by a matrix (i.e the kernel) to produce another vector. Include in your answer a definition of a *stationary* filter, and a *seperable* filter, and explain how these properties simplify the computational work of image filtering.

[8 marks]

(b) Consider the 3×3 stationary filters:

$$A = \begin{vmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{vmatrix} \qquad B = \begin{vmatrix} \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \\ \frac{2}{16} & \frac{4}{16} & \frac{2}{16} \\ \frac{1}{16} & \frac{2}{16} & \frac{1}{16} \end{vmatrix} \quad .$$

Show that both these filters are seperable

[3 marks]

(c) It is proposed to use one of the filters in part b) to smooth an image. By considering an image as the sum of sinusoidal images of different frequencies, discuss the relative performance of each filter in reducing high frequency components in images.

[11 marks]

(d) Suppose that an image is known to have been smoothed by one of the filters in part (b). Discuss how to implement *deconvolution* to recover the original image before smoothing. Why is this process unstable with respect to noise that may have been added after smoothing, and what can be done to control this instability?

[11 marks]

[Total 33 marks]

[CONTINUED]

Question 3

(a) The following equation is often used to describe the formation of a grey-level image by an idealised camera:

$$E(\mathbf{x}) = \frac{\pi D^2 \cos^4(\theta)}{4f^2} L(\theta, \phi, \mathbf{X}) \qquad . \qquad (1)$$

Explain what each of the terms in equation (1) stands for, describe the relationship between $\mathbf{x}$ and $\mathbf{X}$, and explain the physical origin of each of the terms on the right hand side of the equation.

[6 marks]

(b) Define what is meant by the term 'image grey-level histogram' and describe how it is related to the properties of the scene being imaged. Describe qualitatively what you would expect the grey-level histogram for the image in figure 1 below to look like and explain why.

[6 marks]

Figure 1. Image of a portion of the South Downs Way, reproduced in monochrome.

[Question 1 cont. over page]

[TURN OVER]

[Question 3 cont.]

(c) Define what is meant by 'an image segmentation' and explain why segmentation is usually carried out. Explain why it would be difficult to obtain a good segmentation of the image of the South Downs Way shown in figure 1 by use of grey-level thresholds alone.

[6 marks]

(d) Describe a 'region-growing' segmentation algorithm designed, for example, to try to segment the cloud, sky, grass and chalk regions of the image shown in figure 1. Explain why you would not expect such a region-growing algorithm to work well on the image given in figure 1.

[7 marks]

(e) Explain what is meant by a 'region-merging' algorithm and describe how, in principle, such an algorithm works. An adaptive region-merging algorithm uses the measure

$$L = \frac{\sigma_0^{m+n}}{\sigma_i^m \sigma_j^n} \quad , \qquad (2)$$

where the variances σ_i, σ_j and σ_0 are defined over regions P_i, P_j and $P_0 = P_i \cup P_j$ respectively. Explain what the remaining terms used in equation (2) mean and why such a measure must be based on a function of this type irrespective of the underlying statistical model.

[8 marks]

[Total 33 marks]

Question 4

The components on a loaded printed circuit board are inspected automatically by means of a machine vision system which images the boards under controlled lighting conditions as they pass beneath a downwards looking camera mounted above the production line.

(a) The components, which appear bright in the images, are to be isolated by use of grey-level morphological operations followed by application of a suitable threshold. Explain what type of such morphological operations you would use, indicate why you would use them, and describe how they may be defined.

[6 marks]

[Question 2 cont. over page]

[CONTINUE]

[Question 4 cont.]

(b) Explain what is meant by 'template-matching' and describe how, in principle, it may be used to check that components have been correctly placed on the circuit board.

[6 marks]

(c) Describe how L^1 and L^2 norms may be used in template matching and explain why, in practice, use of the L^1 norm might be preferred in the above inspection system.

[5 marks]

(d) Explain how the L^2 norm is related to an 'overlap' or 'cross-correlation' matching measure and describe how the latter may be extended to make it insensitive to: (i) the camera gain or image brightness, and (ii) also to bias of the image intensity. Indicate to what kind of transformations the extended measures in (i) and (ii) are invariant.

[7 marks]

(e) Describe what kinds of errors you would expect the inspection system to make and how, in principle, you would characterise its performance. Explain how, in principle, this characteristic could be used to optimise the system's performance. Briefly indicate what practical problems might make this performance characterization and optimisation difficult.

[9 marks]

[Total 33 marks]

[TURN OVER]

Question 5

(a) Show how homogeneous co-ordinates and a projection matrix may be used to describe the geometry of the formation of an image by an idealised pinhole camera. Indicate how the equations are modified in practice for real cameras and explain why it is advantageous to use the formulation in terms of homogeneous co-ordinates and a projection matrix.

[7 marks]

(b) Describe under what conditions the image formation may be described by:

(i) a Euclidean geometric transformation,

(ii) a similarity transformation,

(iii) an affine transformation,

(iv) must be described by a projective transformation.

[6 marks]

(c) Define the central moments of an image object and explain how normalized central moments may be obtained from them. By considering moments up to second order, describe to what extent and under which of the above geometric transformations such quantities remain invariant.

[7 marks]

(d) Describe how moments may be used for automatic classification of two different types of object, such as leaves of different species of tree that have been imaged, one at a time, on a photographic light-box with a camera mounted vertically above, looking down on the box.

[7 marks]

(e) By using second central moments as an example, illustrate how such an automatic classification system would work for the two species of leaves as described in (d) above, if one type of leaf was much longer and thinner than the other. Indicate whether there would be any advantage to be gained by use of normalized central moments.

[6 marks]

[Total 33 marks]

[END OF PAPER]